Research paper

Hedging longevity risk in the Canadian market

Alain Bélanger and Sullivan Giannini (USherbrooke), and Christian Robert, FCIA

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Abstract

We present some of the results developed in the thesis of Giannini (2020), where an extensive study is performed on the effectiveness of several mortality/longevity-linked financial derivatives in hedging annuity portfolios' longevity risk. We will not treat cashflow hedges like survivor swaps and caps but rather will focus on the value hedges provided by mortality (q-) forwards and puts and Kappa (K-) forwards and puts. We will apply the key q-hedging framework of the seminal paper of Li and Luo (2012) and the key K-hedging outlined in the articles of Chan et al. (2014), Tan et al. (2014) and Li et al. (2019). We will restrict ourselves to static hedging, but our annuity portfolios will be composed of multiple cohorts from the Canadian population. The K-instruments mentioned above are based on the two time-varying indexes of the model proposed by Cairns et al. (2006) and known familiarly as M5.

Preamble

The longevity risk and why bother

The focus on longevity risk is relatively new outside of the insurance industry and has until recently been largely ignored by pension plan sponsors, governments and individuals whose focus was primarily on the management of investment risks. Low interest rates, the aging population and the anemic growth prospects in most developed economies, and the lower expected investment return they entail, have contributed to a greater awareness of the longevity risk and the need for a wider range of solutions through financial markets.

Longevity risk is the danger that we outlive our retirement savings, and the ensuing challenge of maintaining the living standard to which we are accustomed or affording ever-increasing health care and retirement housing costs. Longevity risk also affects all organizations, private or public, which guarantee a lifetime income, death benefits and the reimbursement of long-term health care or funding of social services. As such, individuals, pension plans, insurers and taxpayers may be financially affected by the improvements of survival rates.

Longevity risk differs from investment risk as it is not diversifiable. Private pension plan sponsors wishing to manage their longevity risk have little flexibility and only a few options. They may transfer it to insurers through annuity contracts or enter complex bespoke longevity swap arrangements. Insurers also lack flexibility in effectively managing longevity and mortality risks. New effective risk-hedging solutions would increase their capacity at managing longevity and mortality risk.

Changes to the regulatory framework should also support the utilization of longevity/mortality hedging products. The regulations that pertain to the insurance industry are under review, and as regulators lean towards a greater harmonization globally, we can expect they will promote the use of internal models to manage longevity and mortality risks more actively. The incentive for doing so could include the recognition for such efforts through capital relief, a better appreciation of the risks and more appropriate pricing of life products.

As plan sponsors and insurers turn their attention to longevity risk, solutions incorporating the broader financial market will be essential and will benefit all stakeholders by introducing a more efficient pricing mechanism for longevity and mortality risks.

The appeal of longevity-linked products to investors who are normally not exposed to mortality or longevity risks should stem from the longevity risk premium paid and the low correlation of these risks with any other market risk. This paper is part of an approach to develop effective and practical market solutions to hedge longevity and mortality risks. The development of that market hinges on the two competing requirements of devising a small set of transparent indexes which are yet varied enough so that hedging does not entail too much population basis risk; i.e., the risk of hedging one population's survival with another.

The indexes and hedging instruments presented in this paper are effective in capturing the variability of mortality rates so that only one mortality index would be needed to cover the hedging of annuities or life insurance products for several retirement ages. Moreover, the possibility of developing longevity hedging instruments effective in cross-population settings should enhance their appeal to broad market participation.

1. The CBD models

1.1 CBD-M5

Cairns et al. (2006) proposed the following model for the logit of mortality rates:

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x})$$

where

$q_{x,t}$ is the probability that an individual aged *x* at time *t* dies between *t* and *t* +1;

$\overline{x}$ is the mean of the age range considered (e.g., $\overline{x}$ = 82 for the range 65–99);

$\kappa_t^{(i)}$ for *i* = 1, 2, the time-varying parameters of the model.

The model is calibrated, using Ordinary Least Square (OLS), to the historical mortality rates of Canadian women whose ages range from 65 to 99 for the period 1970–2016. The historical data are provided by the Human Mortality Database (HMD). We have also fitted the parameters by maximum likelihood method (Cairns et al. 2009) and we have obtained practically the same fit.

Once we have the time-varying parameters $K_t^{(1)}$, and $K_t^{(2)}$, we select a VARIMA dynamic that models them jointly. Following Li et al. (2019) we use the bivariate random walk with drift, which is a parsimonious model offering a good fit of the data. More specifically, we have

$$\overrightarrow{\kappa}_t = \overrightarrow{\mu} + \overrightarrow{\kappa}_{t-1} + \mathbb{A}\overrightarrow{z}_t$$

where

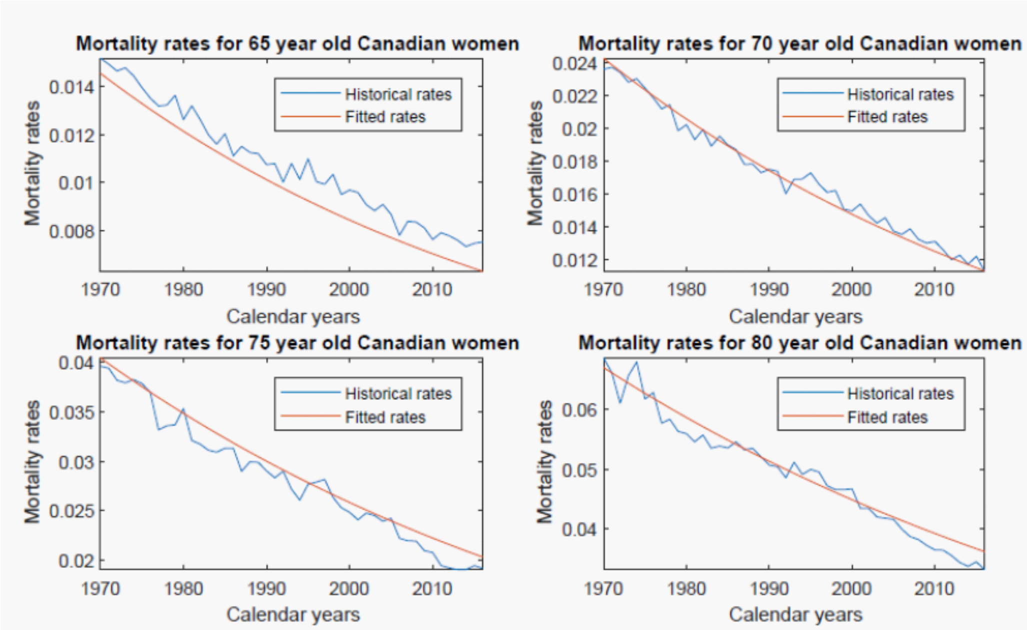
$\overrightarrow{\kappa}_t = (\kappa_t^{(1)}, \kappa_t^{(2)})'$, the time-varying parameters;

$\overrightarrow{\mu} = (\mu_1, \mu_2)'$, the constant drifts;

$\mathbb{A} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}$ the Cholesky decomposition of the covariance matrix of the residuals;

$\vec{z}_t = (z_t^1, z_t^2)'$, independent standard normal error terms.

Here are a few graphical representations of the fit and of the factors for Canadian women of different ages. It is well known that M5 does not provide the best fits of the data. We will compare it later to M7 fits.



1.2 Properties of the indexes

As already mentioned in Chan et al. (2014), the CBD mortality indexes possess the following three desirable properties:

1. Their calculation is transparent and easily interpretable. The first index, $K_t^{(1)}$, represents the general level of the mortality curves all ages. A reduction of $K_t^{(1)}$ implies an overall reduction of mortality. The second index, $K_t^{(2)}$, represents the gradient of the mortality curve. A reduction of $K_t^{(2)}$ implies that mortality at older ages ($x > \bar{x}$) improves more rapidly than at younger ages ($x < \bar{x}$). Education of both indexes for a closed pension plan constitutes a "double whammy" on its liabilities.[1]

[1] We will consider mainly the point of view here of a pension plan hedging its longevity risk, but similar comments can be made for the hedging of mortality risk in a life insurance portfolio. For example, the "double whammy" situation in that case is when there is a reduction of, and an increase of $K_t^{(2)}$. This is important since we do expect to generate a lot of interest from market participants wanting to do these hedging activities as well.

2. The indexes are small in dimension but are able to accurately represent the age patterns of mortality improvement.[2] The possibility of using only two indexes for each population and gender should improve the liquidity of the instruments based on these indexes.[3]
3. The model possesses the new data-invariant property. That is, as new data are considered, the past values of the indexes remain unchanged. Hence maintaining the indexes is easy.

1.3 Alternate model to keep things in check: the CBD-M7 model

As mentioned earlier, we have tested the hedging strategies with Monte Carlo simulations of mortality trajectories generated by M5, the Li–Lee model and M7 (see Giannini 2020). These tests with different types of models give us robust appraisals of the hedging efficiency and, in particular, avoid some type of "model inbreeding" inherent in testing M5 hedges only with M5 generated trajectories. As we will see, M7 offers a much better fit of the data and takes into account cohort effects in particular. We also tested the hedging using the Li–Lee model (applied to the Canadian female and male populations). It does not provide as good a fit as M7, but it is a standard model of the actuarial literature.

M7 is another model proposed by Cairns et al. (2009). The basic equation is:

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \kappa_t^{(3)}\left((x-\overline{x})^2 - \sigma_x^2\right) + \gamma_{t-x}$$

where

σ_x^2 is the mean of $(x-\overline{x})^2$;

$\kappa_t^{(3)}$ is the third time-varying parameter of the model;

γ_{t-x} is the term associated to the cohort born in the year $t-x$.

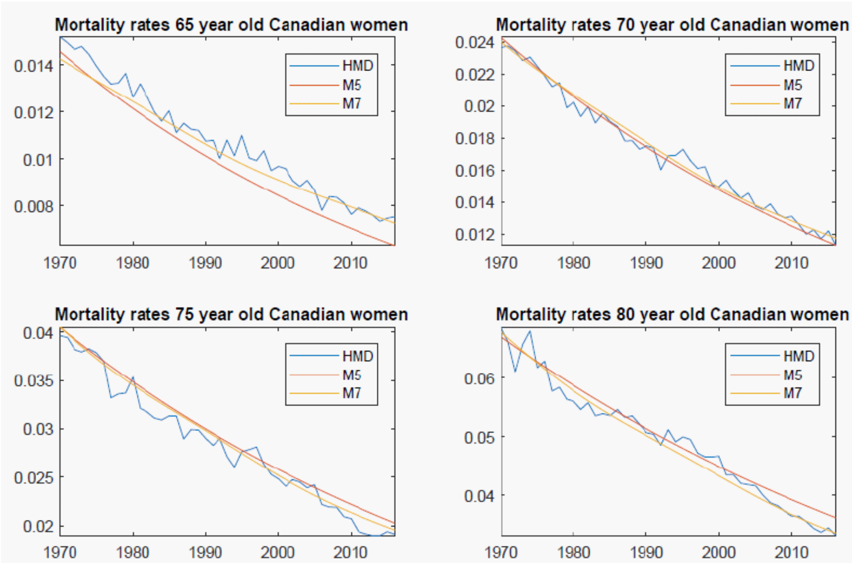
The factors of the model are still easy to compute and easily interpretable: $\kappa_t^{(3)}$ accounts for the curvature of the logit of mortality rates, and γ_{t-x} captures cohort effects. It contains M5 as a nested model: $\kappa_t^{(3)}$ = γ_{t-x} = 0 for all x and t, in the equation above. M5 does not need an indentifiability constraint but we do need to impose the following three identifiability constraints to the cohort effect of M7: $\sum_c \gamma_c = 0; \sum_c c\gamma_c = 0$ and $\sum_c c^2\gamma_c = 0$. Unfortunately, the addition of the cohort effect term (and its constraints), with its dependence on x, implies that the model no longer has the new data-invariant property.

[2] We have tested the hedging efficiency of instruments based on these indexes using M5, the Li–Lee model and M7 to generate mortality trajectories. The three models provide very similar results (see Giannini 2020). With the knowledge that our results are robust, we will only present the results for M7 in this paper.

[3] We have compared the hedging effectiveness of $K_t^{(1)}$ based instruments and $K_t^{(2)}$ based instruments separately (see Giannini 2020). When an instrument is used for hedging, the additional contribution of the corresponding $K_t^{(1)}$ instrument in risk reduction is very marginal. We will therefore focus on $K_t^{(1)}$ instruments only.

But the fit with M7 is greatly improved compared to M5. This is well documented for the populations of England and Wales (Cairns et al. 2009). We will show that the situation is similar for the Canadian population. We will then use M7 to generate mortality rate trajectories taking its factors into account when we test hedging efficiency. Liu and Li (2018) use M7 to study the efficiency of q-forward hedging in the populations of England and Wales.

Graphically at least, M7 seems to offer a better fit than M5:



And this is confirmed with the following statistics of the two fits:

Model	M5	M7
Maximum log-likelihood	-12,047	-9,604.4
Number of parameters	94	222
Number of observations	1,645	1,645
AIC	24,282	19,653
BIC	24,790	20,853
MSE	0.0328	0.0394
MAPE	0.0369	0.0263

Here AIC and BIC denote, as usual, the Akaike and Bayesian information criteria. The only test M7 is not winning is the Mean Square Error (MSE). But, since its Mean Absolute Percentage Error (MAPE) is lower, it means that the M7 fit is hampered by at least one outlier. And indeed, when we do not include the error for the eldest age of 99 (i.e., we restrict the age window to 65–98, say), we get MSEs of 0.0274 and 0.0234 for M5 and M7 respectively.

1.4 Additional calibration of the model parameters to incorporate the risk premium

In order to price all instruments in each of the three models, we have used the Canonical Valuation procedure, outlined in Li and Ng (2011), and we have applied it to the EIB/Paribas longevity bond (Cairns et al. 2009. The bond has a tenor of 25 years and it is linked to the survival of the male population of England and Wales. It was priced with a premium of δ = 0.002 in the equation

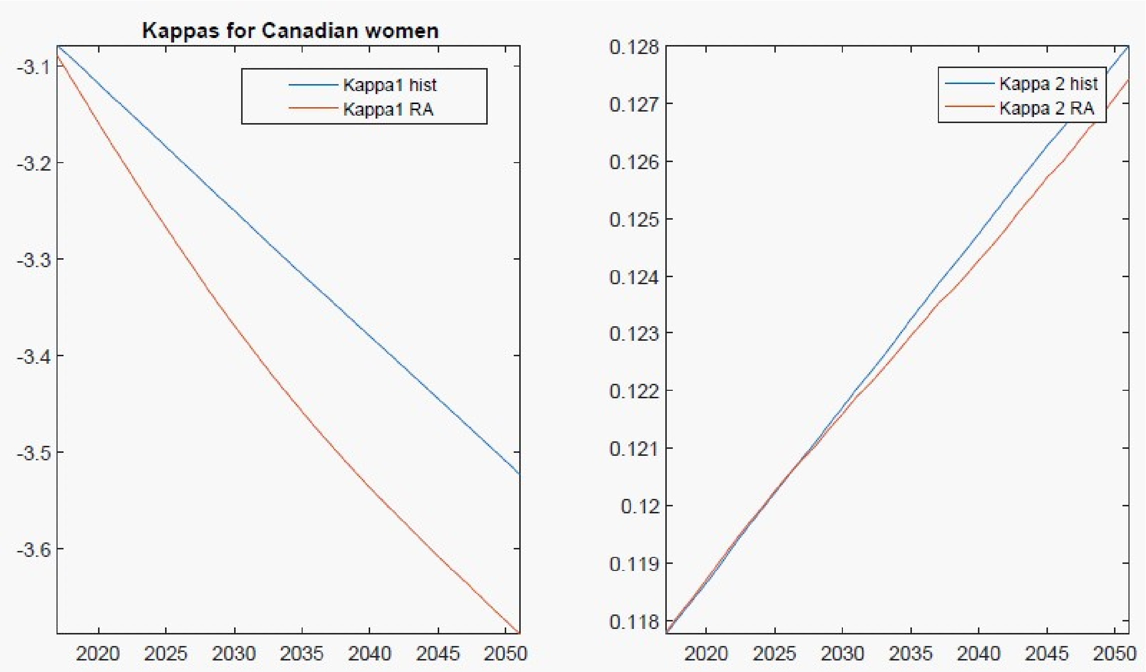
$$P = \sum_{t=1}^{25} B(0,t)\exp(\delta t)\mathbb{E}^{BE}[S_t]$$

where B (0, *t*) is the discount factor for time *t* and $\mathbb{E}^{BE}[S_t]$ is the (Best Estimate) survival forecast of the population at time t in the historical measure.

With any one of the three models, we select a cohort (e.g., Canadian women aged 65) and perform Monte Carlo simulations where we generate 10,000 paths of mortality rates with the model, we value the obligation for each path and we solve for the probabilities that give us the price *P* calculated with the formula above. These probabilities define our risk-adjusted (RA) measure; i.e.,

$$P = \sum_{t=1}^{25} B(0,t)\mathbb{E}^{RA}[S_t].$$

We can then use these probabilities to price any instrument linked to the mortality/longevity of the selected cohort. Mortality (q-) instruments are linked to a single cohort but in the case of K-instruments we use the cohort for the mean age, $\overline{x}$ = 82, of the age window considered, to price the instruments. Here, as an example, are the adjustments made to the expected trajectories for the two Kappa factors of M5.



2. The portfolio of annuities

We consider a synthetic portfolio composed of a total of 4,000 annuitants in 20 cohorts, with 275 annuitants for each of the cohorts aged 65–69, 200 annuitants for each aged 70–74, 175 for each aged 75–79, and 150 for each aged 80–84. The annuities are immediate whole life annuities which pay an amount of $1 to all surviving individuals. For the purpose of this study, we have focused on Canadian women cohorts. If *k* = 1:4000 denotes an annuitant, let a_k denote her age at time 0. The assets of the pension plan are calculated using the RA measure:

$$A = \mathbb{E}^{RA}\left[\sum_{k=1}^{4000}\sum_{t=1}^{\omega - a_k} B(0,t)S_{a_k}(t)\right]$$

where $S_{a_k}(t)$ is the survival rate for age a_k ; $B(0,t)$ is the discount rate to time *t*, and *w* is the maximum age considered, which we assume to be 100 years. Each annuitant, *k*, generates a sequence of liabilities $L_k = \sum_{t=1}^{\lfloor \tau_k \rfloor \wedge \omega} B(0,t)$, where τ_k is the time of death of the k^{th} annuitant. The quantity to hedge is the surplus $D = A - \sum_{k=1}^{4000} L_k$, which is convenient to consider per contract: $\overline{D} = D/4000$. If *H* denotes the discounted payoff of the hedging portfolio then we will look at the distribution of $\overline{D}_F = (D + H_F)/4000$ when we hedge with forwards and $\overline{D}_P = (D + H_P - premium)/4000$ when we edge with puts.

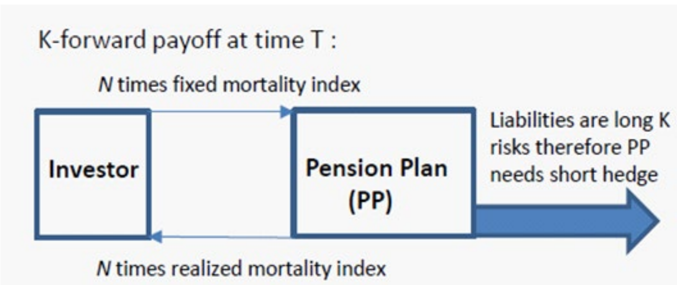
Each Monte Carlo simulation we perform to test the hedges involves generating the time of death, τ_k, of each annuitant. With these realizations we calculate the liability payoff as well as the payoffs of the hedging instruments. We then compare the descriptive statistics of the distributions to assess the hedges' effectiveness. The times of death are generated as the first jump of a Cox process whose mortality trajectories are generated by realizations from M7 (M5 or Li–Lee).

3. The hedging instruments

We introduce the K-instruments first. We will compare their hedging efficiency with that of the mortality instruments which we describe afterwards.

3.1 K-forwards

Let *N* denote the notional of the transaction:



Let t_0 denote the time at which the forward is initiated and let *T* denote its maturity. Let *N* = $1 for simplicity. The payoff from PP zczc perspective is $\mathcal{F}^{(i)}(T,K) = K^{(i)} - \kappa_T^{(i)}$ where $K^{(i)}$ is the fixed mortality index and $\kappa_T^{(i)}$ is the realized index at time *T* with $i = 1, 2$. Its value at time $t : t_0 \leq t < T$ is

$$\begin{aligned} F_t^{(i)}(T, K) &= (1 + r_f)^{-(T-t)} \mathbb{E}_t^{RA} \left[\mathcal{F}^{(i)}(T, K^{(i)}) \right] \\ &= (1 + r_f)^{-(T-t)} \left(K^{(i)} - \mathbb{E}_t^{RA} \left[\kappa_T^{(i)} \right] \right) \end{aligned}$$

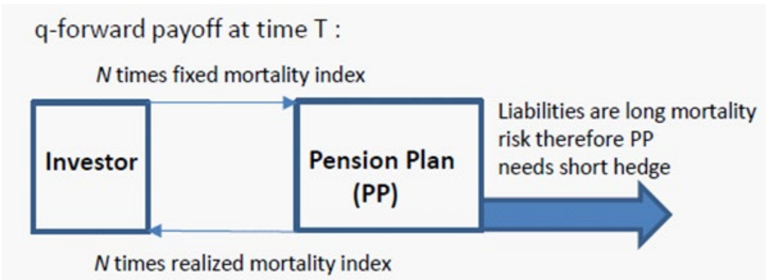
As usual, we do not want an upfront payment to be made, so $F_{t_0}^{(i)}(T, K) = 0$ and as a result $K^{(i)} = \mathbb{E}_{t_0}^{RA} \left[\kappa_T^{(i)} \right]$. As we mentioned earlier, we will only consider the instruments based on $\kappa_t^{(1)}$, since they are ones that matter, and we will let $K = \mathbb{E}_{t_0}^{RA} \left[\kappa_T^{(1)} \right]$. In order to simplify notations, the letter *K* will be used throughout to indicate the level of the fixed side and the exercise level of the puts for all the hedging instruments. Its meaning should be clear in the context of each instrument.

3.2 K-puts

The payoff for the long position is $\mathcal{P}^{(1)}(T, K) = (K - \kappa_T^{(1)})^+$ and its value (premium) at time $t < T$ is $P^{(1)}(T, K) = (1 + r_f)^{-(T-t)} \mathbb{E}_t^{RA} \left[(K - \kappa_T^{(1)})^+ \right]$. We will always assume (in this paper) that $K = \mathbb{E}_{t_0}^{BE} \left[\kappa_T^{(1)} \right]$ and we will say that the put is At-The-Money (ATM).[4]

We now introduce the mortality instruments.

3.3 q-forwards



Again, let *N* = \$1. So the payoff from PP zczc perspective is $q\mathcal{F}(T, K) = K - q_{x,T}$, where *K* is the fixed mortality rate and $q_{x,T}$ is the realized mortality for age *x* during year *T*. Its value at time *t* : $t : t_0 \leq t < T$ is

$$\begin{aligned} qF(T, K) &= (1 + r_f)^{-(T-t)} \mathbb{E}_t^{RA} \left[q\mathcal{F}(T, K) \right] \\ &= (1 + r_f)^{-(T-t)} \left(K - \mathbb{E}_t^{RA} \left[q_{x,T} \right] \right) \end{aligned}$$

As usual, we do not want an upfront payment to be made, so $qF_{t_0}(T, K) = 0$ and $K = \mathbb{E}_{t_0}^{RA} \left[q_{x,T} \right]$.

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[4] We will focus here on puts and portfolio insurance strategies. It is also possible to hedge the portfolio by shorting calls, but the covered call strategy is only deployed when small movements of mortality rates are anticipated as it provides only a small cushion against lower mortality rates.

3.4 q-Puts

The payoff for the long position is $qP(T,K) = (K - q_{x,T})^+$ and its value (premium) at time $t : t_0 < t < T$ is $qP(T,K) = (1+r_f)^{-(T-t)}\mathbb{E}_t^{RA}\left[(K - q_{x,T})^+\right]$. Again, we will let $K = \mathbb{E}_{t_0}^{BE}\left[q_{x,T}\right]$ and we will say that the put is ATM.

4. Sensitivities and key q-durations, and key K-durations hedges

The concept of key q-duration is due to Li and Luo (2012). The idea as described therein is to use the correlation of mortality rates for adjacent ages and adjacent years in order to select interspersed years and ages that capture the sensitivities of the portfolio to movements of the mortality rate surface. The sensitivities are calculated with the M7 model. Very similar results are obtained in Giannini (2020) for the Li–Lee model and M5. We apply Li and Luo's method directly to derive the following q-hedging instruments:

Cohort k	Age in 2017	n_k	Age and maturity	Notional w(j,k)
1	68	1	(70, 2019)	19.0100
		2	(75, 2024)	17.7177
		3	(80, 2029)	13.0434
		4	(85, 2034)	8.4472
		5	(90, 2039)	4.4354
2	72	1	(75, 2020)	9.2600
		2	(80, 2025)	7.6359
		3	(85, 2030)	4.8908
		4	(90, 2035)	2.5296
3	76	1	(80, 2021)	6.6702
		2	(85, 2026)	4.4962
		3	(90, 2031)	2.3113
4	80	1	(85, 2022)	4.2232
		2	(90, 2027)	2.1773

In all our mortality hedging portfolios, we will always take all the key instruments (n_k) associated with a cohort. Hence, we can index the portfolios by the cohort number only. So, we can denote by F_1, for instance, the portfolio of forwards {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5)}. Similarly, $F_{1,2}$ will denote the set of forwards {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4)}; etc. We will use the notation P_1, $P_{1,2}$, etc., similarly for the portfolios of puts.

The concept of key K-duration, inspired by the key q-duration of Li and Luo (2012), is developed in Tan et al. (2014) and then further investigated in Li et al. (2019). As mentioned, Giannini (2020) showed that hedging instruments based on $K_t^{(2)}$ were very marginally contributing to the risk reduction, so we will focus only on $K_t^{(1)}$ instruments for the following key durations. Again, the sensitivities shown here are calculated with M7, but Giannini (2020) showed very similar results for M5 and Li–Lee:

	j = 1	j = 2	j = 3	j = 4
Key year	2022	2027	2032	2037
	-0.07903	-0.6496	-0.5339	-0.4388
	-0.8988	-0.7722	-0.5817	-0.3679
W_j K1	1.1373	1.1887	1.0894	0.8385

5. Static hedging with mortality and K-instruments

5.1 Mortality (q-) hedging

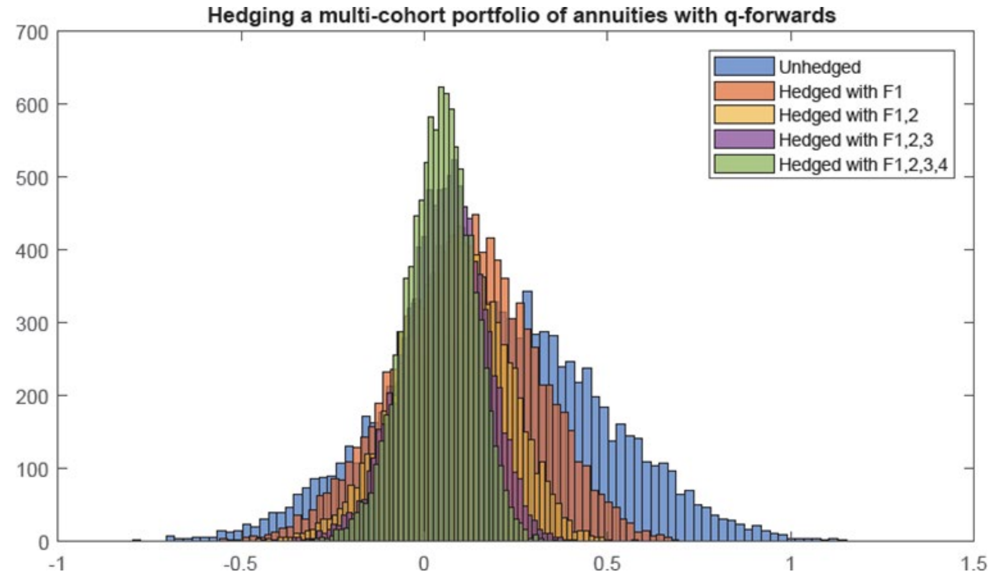
In order to test the efficiency of the key q-hedging method with forwards and puts (separately), we used M7 to generate mortality trajectories and calculated the descriptive statistics of the distributions of the payoffs of the annuity portfolio when incremental sets of hedging instruments were used. Very similar results were obtained for M5 and Li–Lee (see Giannini 2020). These descriptive statistics were first used by Fung et al. (2019) to test the efficiency of survivor (s-) swaps and caps.

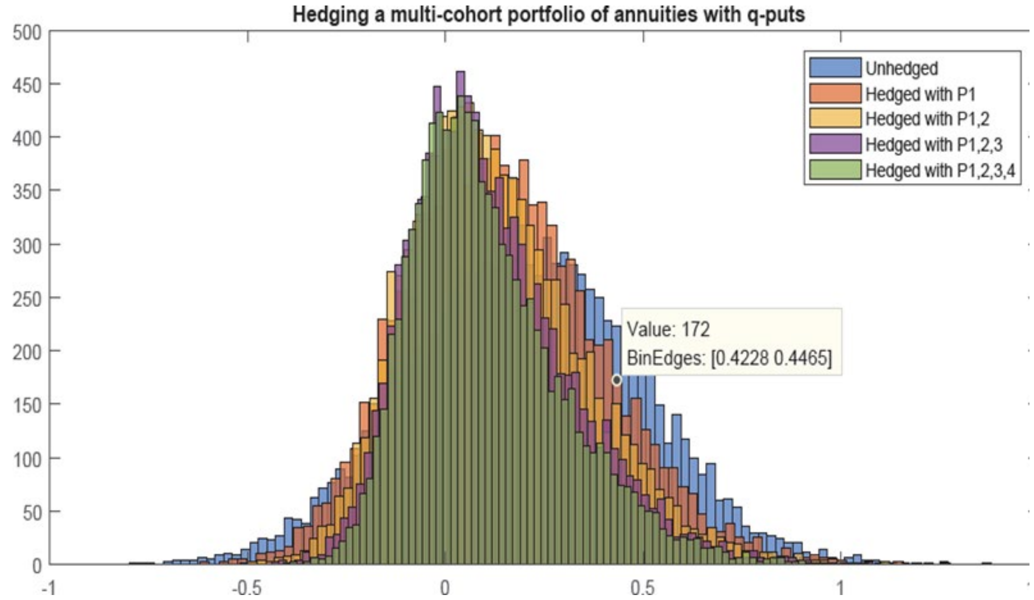
Portfolio	Mean	Std. Dev.	Skewness	Kurtosis	VaR99	ES99
Unhedged	0.1976	0.2880	0.0314	2.9281	-0.4754	-0.5471
F_1	0.1144	0.1928	-0.1083	2.9093	-0.3529	-0.4072
$F_{1,2}$	0.0782	0.1446	-0.2002	2.9614	-0.2841	-0.3269
$F_{1,2,3}$	0.0554	0.1108	-0.2561	3.0538	-0.2297	-0.2624
$F_{1,2,3,4}$	0.0428	0.0921	-0.2413	3.1240	-0.1906	-0.2234

Portfolio	Mean	Std. Dev.	Skewness	Kurtosis	VaR99	ES99
Unhedged	0.1976	0.2880	0.0314	2.9281	-0.4754	-0.5471
P_1	0.1473	0.2433	0.3401	3.1313	-0.3694	-0.4234
$P_{1,2}$	0.1261	0.2205	0.5533	3.3791	-0.3106	-0.3537
$P_{1,2,3}$	0.1132	0.2039	0.7382	3.6741	-0.2652	-0.2990
$P_{1,2,3,4}$	0.1062	0.1930	0.8650	3.9303	-0.2307	-0.2649

Where VaR99 and ES99 mean the Value-at-Risk and Expected Shortfall both at a 99% confidence level.

If we were to measure the efficiency of the hedges in terms of a reduction of the payoffs standard deviation, we would conclude overwhelmingly that q-forward hedges are superior to put hedges. But this measure ignores other features/moments of the distribution, like its skewness in particular. If we look instead at the reductions in VaR and Expected Shortfall we get a more balanced conclusion on the hedging efficiency of each instrument. Overall, the full hedging portfolios with 14 q-forwards or q-puts reduce the Expected Shortfall by more than half. Also, hedging with puts is less expensive than with forwards even though our puts are all ATM.


Hedging a multi-cohort portfolio of annuities with q-forwards
Unhedged
Hedged with F1
Hedged with F1,2
Hedged with F1,2,3
Hedged with F1,2,3,4
700
600
500
400
300
200
100
0
-1
-0.5
0
0.5
1
1.5


Hedging a multi-cohort portfolio of annuities with q-puts
Unhedged
Hedged with P1
Hedged with P1,2
Hedged with P1,2,3
Hedged with P1,2,3,4
Value: 172
BinEdges: [0.4228 0.4465]
500
450
400
350
300
250
200
150
100
50
0
-1
-0.5
0
0.5
1

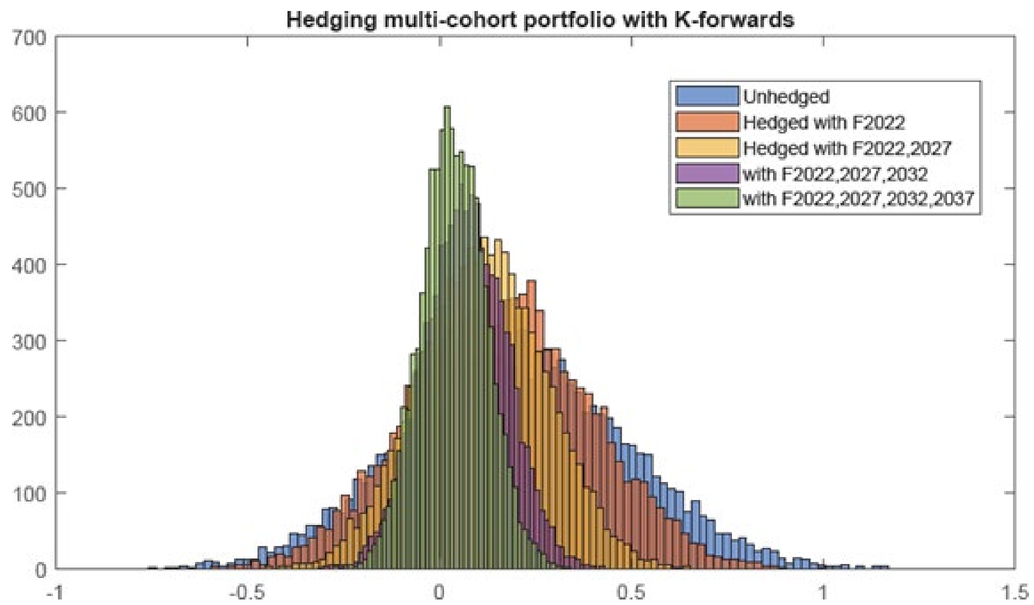
5.2 K-hedging

We will now test K-hedging. Again the results obtained from the three models are very similar. Here are the results for M7:

Portfolio	Mean	Std. Dev.	Skewness	Kurtosis	VaR99	ES99
Unhedged	0.1946	0.2922	0.0153	2.9387	-0.4748	-0.5617
F_{2022}	0.1569	0.2355	0.0105	2.9254	-0.3865	-0.4534
$F_{2022, 2027}$	0.1078	0.1644	-0.0011	2.9614	-0.2697	-0.3183
$F_{2022, 2027, 2032}$	0.0638	0.1985	-0.0123	3.0538	-0.1856	-0.2178
$F_{2022, 2027, 2032, 2037}$	0.0335	0.0850	-0.0391	3.1240	-0.1607	-0.1856

Portfolio	Mean	Std. Dev.	Skewness	Kurtosis	VaR99	ES99
Unhedged	0.1946	0.2922	0.0153	2.9387	-0.4748	-0.5617
P_{2022}	0.1722	0.2636	0.1977	3.0065	-0.4027	-0.4692
$P_{2022, 2027}$	0.1423	0.2280	0.5206	3.3173	-0.3020	-0.3504
$P_{2022, 2027, 2032}$	0.1153	0.1985	0.8666	3.9012	-0.2271	-0.2618
$P_{2022, 2027, 2032, 2037}$	0.0967	0.1807	1.0793	4.4658	-0.1971	-0.2265

Compared with the q-hedges, the full K-hedges provide an even greater reduction of both standard deviation and Expected Shortfall. This is remarkable since it is achieved with only four instruments as opposed to 14. We note again the lower costs of the ATM K-put hedges:



Here is a summary of percentage reductions of the Expected Shortfall for each type of hedges in the three models:

Cohorts		**1**	**1, 2**	**1, 2, 3**	**1, 2, 3, 4**
Li–Lee	Q-forwards	28.86%	44.64%	56.19%	63.15%
	Q-puts	25.67%	39.46%	49.31%	55.32%
CBD (M5)	Q-forwards	25.77%	40.20%	51.51%	58.87%
	Q-puts	22.69%	35.04%	44.51%	50.50%
CBD (M7)	Q-forwards	25.57%	40.25%	52.04%	59.17%
	Q-puts	22.61%	35.35%	45.35%	51.58%

Year		**2022**	**2022, 2027**	**2022, 2027, 2032**	**2022, 2027, 2032, 2037**
Li–Lee	K1-forwards	23.00%	48.27%	63.38%	62.75%
	K1-puts	20.10%	42.49%	56.39%	60.36%
CBD (M5)	K1-forwards	19.52%	43.12%	60.48%	65.46%
	K1-puts	16.71%	37.48%	53.20%	59.50%
CBD (M7)	K1-forwards	19.28%	43.33%	61.22%	66.96%
	K1-puts	16.47%	37.62%	53.39%	59.68%

Each individual table shows some variability across models for each portfolio. Some of it is due to the Monte Carlo simulations of course and could be dealt with using the same random numbers for each model or using more simulations and variance-reduction techniques. But the general picture these numbers give in terms of hedging efficiency is fairly consistent.

Understandably, the K-hedge with only one instrument offers a lower reduction than the five q-forwards but it is quite remarkable to see the reductions provided by the K-instruments improving with the incremental addition of a single K-instrument, to the point where the reductions provided by the hedges with three and four K-instruments are all higher than the reductions of the corresponding 12 and 14 q-instruments respectively.

6. Conclusion

Our study shows that mortality and K-forwards and puts provide efficient means of greatly reducing longevity risk in a multi-cohort portfolio of annuities.

It would be very useful for pension plans and insurance companies to have exchanges where they could take short positions on q-futures and K-futures as well as long positions on q-puts and K-puts to hedge their longevity risk. On the other side of these transactions, we believe that capital market investors (besides mortality hedgers like insurance and re-insurance companies) would be greatly interested in taking opposite positions in these instruments in order to earn a premium for a risk which is largely uncorrelated with their market risks. Among those products (and along with the survivor instruments treated in Giannini 2020), the K-futures and puts – without a need for a specific reference to age, and with key-durations five years apart, say – offer excellent hedges requiring only a few instruments to be traded in the market. This means more liquid instruments and less costly portfolio rebalancing. If we add to this the ease and transparency of computation of the indices, their intuitive interpretation in terms of movements of mortality rates, and the new data-invariant property which makes the indices easy to maintain, we get a very compelling case for the rapid development of a liquid K-futures/puts market. In the case of the puts, the hedger also has the opportunity of choosing the deductible on the portfolio insurance. This cost efficiency vs moneyness of the options is the main theme of Li et al. (2019) and it is something we want to investigate later, along with the efficiency of dynamic hedging. Zhou and Li (2019) study delta-hedging cross-population basis risk with q-forwards. Dynamic hedging requires a well-developed and liquid market.

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