Member’s Paper

A Comparative Analysis of PfAD Designs

By
Chun-Ming (George) Ma, FCIA

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Abstract	**Résumé**
Several Canadian pension jurisdictions have revised their funding regulations for defined benefit pension plans in recent years, aiming to eliminate or ease the solvency funding requirement while strengthening the going-concern funding requirement by adding a provision for adverse deviation (PfAD). There are fundamental differences in the PfAD designs among those jurisdictions. This paper presents a comparative analysis of three known PfAD design alternatives, with a focus on their effectiveness at stabilizing funding for pension plans under a changing-interest-rate environment.	Ces dernières années, plusieurs juridictions chargées des retraites au Canada ont mis à jour leur réglementation sur le provisionnement des régimes à prestations déterminées. L'objectif est d'éliminer ou de diminuer les exigences de provisionnement de solvabilité, tout en augmentant les exigences de provision de passifs à long terme avec l'ajout d'une provision pour écarts défavorables (PED). D'une juridiction à l'autre, le design des PED varie énormément. Le présent document fait l'analyse comparative de trois designs de PED distincts et, plus particulièrement, de leur efficacité à stabiliser le provisionnement des régimes de retrait dans une période de variation des taux d'intérêt.

1. Introduction

In our paper on funding margins published by the CIA (CIA Member's Paper) (Ma, 2018), we raised concerns about the provision for adverse deviations (PfAD) included in the funding requirements for defined benefit pension plans in Ontario and Québec. Concurrently, we proposed an alternative methodology for developing a dynamic discount rate margin to better address the funding risk arising from interest rate changes.

More recently, British Columbia implemented major changes to the funding requirements for pension plans effective December 31, 2019 (BCFSA, 2020). The new regulations require the going-concern funding target for a defined benefit plan to include a PfAD designed to reduce long-term interest rate risk. Unlike Ontario and Québec, the PfAD adopted in BC does not depend on the plan's investment policy or maturity.

This paper is written as an extension of the CIA Member Paper. We present a modelling of an open plan to assess the Ontario and BC approaches, and our proposed PfAD approach, regarding how effective they are in stabilizing funding for pension plans under a changing-interest-rate environment.

2. Ontario PfAD Design

Consider an open plan where 50% of the plan's liabilities are related to members with pensions in pay. The plan adopts an investment policy with a target asset mix of 60% equities and 40% fixed income. According to the Ontario regulations, the plan must fund for a PfAD equal to 8% of the liabilities of the plan, calculated using a benchmark discount rate (BDR). The BDR for this plan is determined by the following formula prescribed in the regulations:

0.005 + H + (0.015 × 0.4) + (0.05 × 0.6)

where H is the yield on long-term Government of Canada (GoC) bonds (CANSIM series 39056) for the valuation date. The value of 0.005 in this formula represents an excess return from diversification and rebalancing of the pension fund. For our analysis, we assume that this value is only valid for plans with a target asset mix of 50% equities and 50% fixed income, and use instead an adjusted value of 0.004 for the model plan. As at January 1, 2020, the long-term GoC bond yield (annualized) is 1.46%. The BDR for the valuation at this date is equal to: 0.004+0.0146+0.006+0.03 = 0.0546 or 5.46%.

If the plan's target asset mix is maintained, the BDRs would move in tandem with the long-term GoC bond yields.

Back-testing

We apply the Ontario PfAD rule to the plan over the 20-year period starting on January 1, 2000, assuming a valuation of the plan is performed at each anniversary.

First, we introduce some notation:

i_t	BDR applied at time t
$L_{t,s}$	Liabilities of the plan as of time t calculated using a BDR of i_s
δ_t	Adjustment factor applied to $L_{t,t-1}$ such that $L_{t,t} = (1 + \delta_t) \cdot L_{t,t-1}$
$PfAD_t$	PfAD applied at time t

Consider two consecutive valuation dates, time 0 and time 1. Had there been no change in the BDR and PfAD, the liabilities expected to be funded at time 1 would be equal to: $(1 + PfAD_0) \cdot L_{1,0}$, where $L_{1,0}$ is the liabilities determined at time 1 using a BDR of i_0. If the BDR at time 1 is changed to i_1 (due to a change in the GoC bond yield) and the PfAD is changed to $PfAD_1$, the increase (decrease) in liabilities required to be funded at time 1 would be as follows:

$$\Delta L_1 = (1 + PfAD_1)L_{1,1} - (1 + PfAD_0)L_{1,0} \quad (1)$$

Since $PfAD_1 = PfAD_0 = 0.08$ and $L_{1,1} = (1 + \delta_1)L_{1,0}$, it follows that:

$$\Delta L_1 = 1.08(1 + \delta_1)L_{1,0} - 1.08L_{1.0} = 1.08 \cdot \delta_1 \cdot L_{1,0} \quad (2)$$

At any time t, the increase (decrease) in liabilities required to be funded, due to a change in the GoC bond yield, can be written as follows:

$$\Delta L_t = 1.08 \cdot \delta_t \cdot L_{t,t-1}$$

ΔL_t, expressed as a percentage of $1.08 \cdot L_{t,t-1}$, is therefore equal to δ_t.

Note that $\delta_t = \frac{L_{t,t}}{L_{t,t-1}} - 1$. The liabilities $L_{t,t}$ (based on BDR of i_t) can be estimated from the liabilities $L_{t,t-1}$ (based on BDR of i_{t-1}) using the following formula developed in a paper on discount rate sensitivities published by the CIA and the Society of Actuaries (SOA, 2017):

$$L_{t,t} = exp\left[-(18 - 10.5p) \times (i_t - i_{t-1}) \times \left(1 - 8\left(\frac{i_{t-1} + i_t}{2} - 5.25\%\right)\right)\right] \times L_{t,t-1}$$

where p is the proportion of the plan liabilities that relate to pensions in pay.

For the model plan, $p = 0.5$. It follows that:

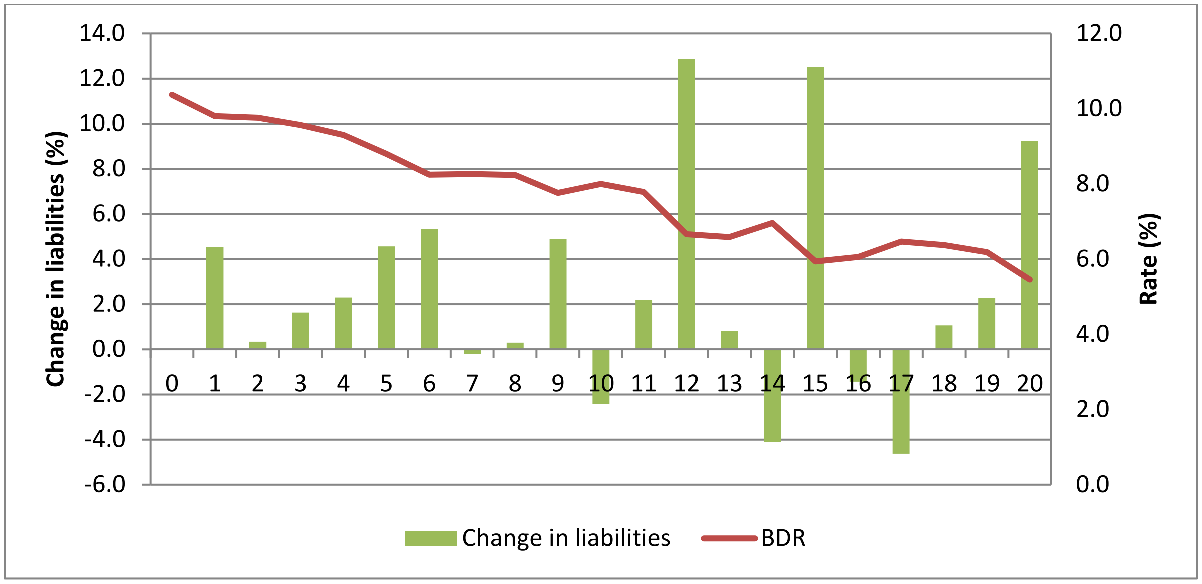
$$\delta_t = exp\left[-(18 - 10.5 \times 0.5) \times (i_t - i_{t-1}) \times \left(1 - 8\left(\frac{i_{t-1} + i_t}{2} - 5.25\%\right)\right)\right] - 1 \qquad (3)$$

Table 1 shows the percentage increase (or decrease) in liabilities required to be funded, due solely to a change in the long-term GoC bond yield, at each valuation date between January 1, 2001, and January 1, 2020 inclusive.

Table 1 – Ontario PfAD Design

Valuation Date	Time	GoC Bond Yield (Annualized %)	BDR (%)	PfAD (%)	Increase (Decrease) in Liabilities (%)
2000 Jan	0	6.37	10.37	8.00	
2001 Jan	1	5.80	9.80	8.00	4.53
2002 Jan	2	5.76	9.76	8.00	0.33
2003 Jan	3	5.57	9.57	8.00	1.62
2004 Jan	4	5.30	9.30	8.00	2.29
2005 Jan	5	4.80	8.80	8.00	4.56
2006 Jan	6	4.24	8.24	8.00	5.33
2007 Jan	7	4.26	8.26	8.00	(0.20)
2008 Jan	8	4.23	8.23	8.00	0.30
2009 Jan	9	3.75	7.75	8.00	4.88
2010 Jan	10	4.00	8.00	8.00	(2.43)
2011 Jan	11	3.79	7.79	8.00	2.18
2012 Jan	12	2.66	6.66	8.00	12.87
2013 Jan	13	2.59	6.59	8.00	0.81
2014 Jan	14	2.96	6.96	8.00	(4.11)
2015 Jan	15	1.94	5.94	8.00	12.50
2016 Jan	16	2.06	6.06	8.00	(1.44)
2017 Jan	17	2.47	6.47	8.00	(4.63)
2018 Jan	18	2.37	6.37	8.00	1.06
2019 Jan	19	2.18	6.18	8.00	2.27
2020 Jan	20	1.46	5.46	8.00	9.24

The funding impact due to changes in GoC bond yields can be shown graphically as follows:



As shown in Table 1, the percentage increase in liabilities to be funded at January 1, 2020, is 9.24%. This is estimated using Equation (3) with $i_{19} = .0618$ and $i_{20} = .0546$:

$$\delta_{20} = exp\left[-(18 - 10.5 \times 0.5) \times (.0546 - .0618) \times \left(1 - 8 \times \left(\left(\frac{.0546 + .0618}{2}\right) - .0525\right)\right)\right] - 1 = .0924$$

Observations

The Ontario PfAD design does not address a funding risk that concerns most defined benefit plan sponsors. That risk is the unexpected increase in required contributions resulting from a decline in long-term GoC bond yields upon which the BDR is based.

The GoC bond yields have been trending downward since year 2000. Except for January 2007, 2010, 2014, 2016 and 2017, there were decreases in GoC bond yields in other years. The PfAD (fixed at 8% for the model plan) does not reduce the funding impact due to a decline in interest rates. Rather, it magnifies the liabilities required to be funded (by a factor of 1.08 for this plan) as GoC bond yields decline.

Unlike Ontario, Québec's PfAD is derived from a two-dimensional grid based on a plan's target asset mix and level of mismatch in asset and liability duration. The PfAD for a plan would still be a fixed percentage if the plan's asset mix and the ratio of asset to liability duration are held constant. As such, the funding pattern under Québec's PfAD approach would be similar to that of Ontario's over the 20-year period.

Set out below are some statistics on the percentage increase (decrease) in liabilities required to be funded over the 20-year period. The percentage funding increase, due solely to a decline in the GoC bond yield, could be as high as 12.9%.

Mean	2.60%
Standard Deviation	4.77%
Maximum	12.87%
Minimum	(4.63%)

3. Alternative PfAD Methodology

In the CIA Member Paper, we proposed an alternative design for a PfAD through developing a margin for incorporation in the going-concern discount rate (GCDR). The margin reflects a plan's investment policy, its level of maturity and the current level of long-term interest rates. It moves with long-term interest rates that fall within a specified range. We have shown that the GCDR incorporating such a dynamic margin would be more stable than the BDR. This has the effect of stabilizing the funding requirement in a volatile interest rate environment, as demonstrated below.

Back-testing

Again, we apply the alternative PfAD methodology to the above plan over a 20-year period starting on January 1, 2000. For this analysis, we introduce some additional notation:

$\widetilde{\imath_t}$	GCDR applied at time t
$\widetilde{L_{t,s}}$	Liabilities of the plan as of time t calculated using GCDR of $\widetilde{\imath_s}$
$\widetilde{\delta_t}$	Adjustment factor applied to $\widetilde{L_{t,t-1}}$ such that $\widetilde{L_{t,t}} = (1+\widetilde{\delta_t}) \cdot \widetilde{L_{t,t-1}}$
ρ_t	Adjustment factor applied to $L_{t,t}$ such that $\widetilde{L_{t,t}} = (1+\rho_t) \cdot L_{t,t}$
$\widetilde{PfAD_t}$	PfAD applied at time t under the alternative methodology

The GCDR at a time t, $\widetilde{\imath_t}$, is determined according to the procedure described in the CIA Member Paper. For the plan in our back-testing, it is calculated as the sum of 3.22% (risk premium plus diversification return) and the corresponding "lower bound" rate in Appendix B of the paper.

By definition,

$$\widetilde{PfAD_t} = \rho_t$$

where ρ_t can be estimated using the following formula:

$$\rho_t = exp\left[-(18 - 10.5 \times 0.5) \times (\widetilde{\imath_t} - i_t) \times \left(1 - 8\left(\frac{i_t + \widetilde{\imath_t}}{2} - 5.25\%\right)\right)\right] - 1 \qquad (4)$$

The adjustment factor $\widetilde{\delta_t}$ can be similarly estimated:

$$\widetilde{\delta_t} = exp\left[-(18 - 10.5 \times 0.5) \times (\widetilde{\iota_t} - \widetilde{\iota_{t-1}}) \times \left(1 - 8\left(\frac{\widetilde{\iota_{t-1}} + \widetilde{\iota_t}}{2} - 5.25\%\right)\right)\right] - 1 \tag{5}$$

Consider two consecutive valuation dates, time 0 and time 1. Had there been no change in the GCDR, the liabilities expected to be funded at time 1 would be equal to $\widetilde{L_{1,0}}$. If the GCDR at time 1 is changed to $\widetilde{\iota_1}$, the increase (decrease) in liabilities required to be funded at time 1 would be:

$$\widetilde{\Delta L_1} = \widetilde{L_{1,1}} - \widetilde{L_{1,0}} = \widetilde{\delta_1} \cdot \widetilde{L_{1,0}}$$

Likewise, at any time t, the increase (decrease) in liabilities required to be funded, due to a change in the GCDR, can be determined as follows:

$$\widetilde{\Delta L_t} = \widetilde{\delta_t} \cdot \widetilde{L_{t,t-1}}$$

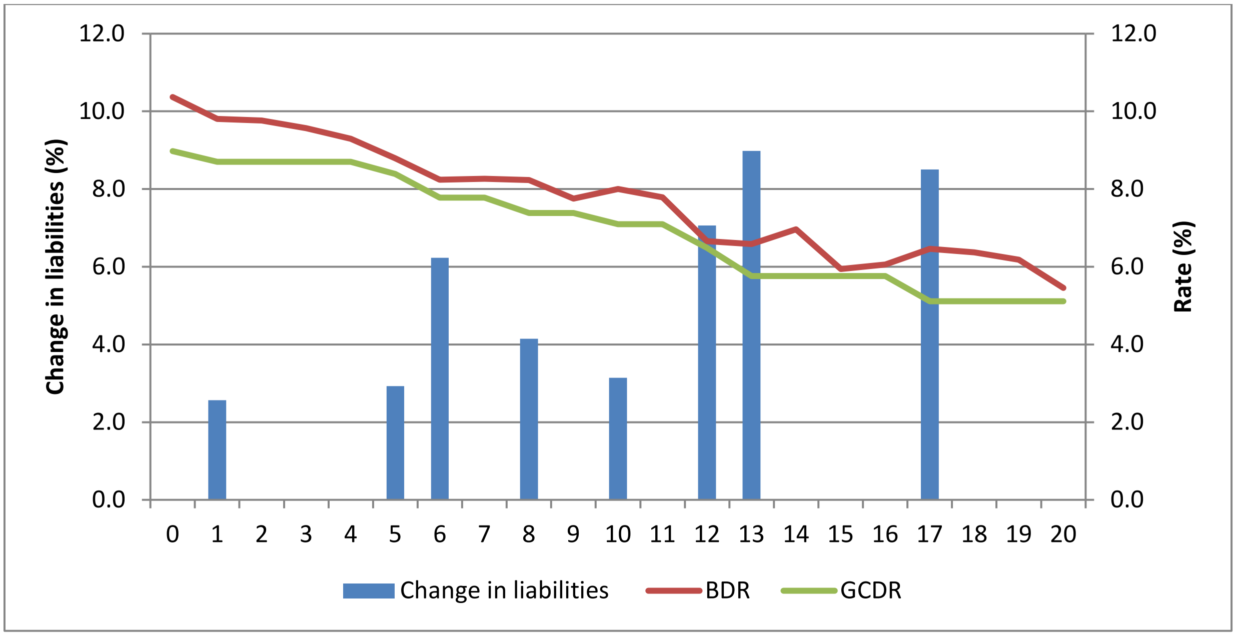
$\widetilde{\Delta L_t}$, expressed as a percentage of $\widetilde{L_{t,t-1}}$, is therefore equal to $\widetilde{\delta_t}$.

Table 2 shows the percentage increase (or decrease) in liabilities required to be funded, due solely to a change in the GCDR, at each valuation date between January 1, 2001, and January 1, 2020 inclusive.

Table 2 – Alternative PfAD Methodology

Valuation Date	Time	BDR (%)	GCDR (%)	PfAD (%)	Increase (Decrease) in Liabilities (%)
2000 Jan	0	10.37	8.98	12.15	
2001 Jan	1	9.80	8.70	10.04	2.56
2002 Jan	2	9.76	8.70	9.67	0.00
2003 Jan	3	9.57	8.70	7.92	0.00
2004 Jan	4	9.30	8.70	5.50	0.00
2005 Jan	5	8.80	8.39	3.86	2.93
2006 Jan	6	8.24	7.78	4.74	6.23
2007 Jan	7	8.26	7.78	4.95	0.00
2008 Jan	8	8.23	7.39	8.98	4.14
2009 Jan	9	7.75	7.39	3.90	0.00
2010 Jan	10	8.00	7.10	9.84	3.14
2011 Jan	11	7.79	7.10	7.50	0.00
2012 Jan	12	6.66	6.49	1.96	7.07
2013 Jan	13	6.59	5.76	10.23	8.98
2014 Jan	14	6.96	5.76	14.96	0.00
2015 Jan	15	5.94	5.76	2.18	0.00
2016 Jan	16	6.06	5.76	3.67	0.00
2017 Jan	17	6.47	5.11	17.95	8.50
2018 Jan	18	6.37	5.11	16.72	0.00
2019 Jan	19	6.18	5.11	14.12	0.00
2020 Jan	20	5.46	5.11	4.46	0.00

The funding impact due to changes in GCDRs can be shown graphically as follows:



The PfAD applied at January 1, 2020, is 4.46% of the liabilities calculated using a BDR of 5.46%. This is estimated using Equation (4) with $i_{20} = .0546$ and $\widetilde{i_{20}} = .0511$:

$$\rho_{20} = exp\left[-(18 - 10.5 \times 0.5) \times (.0511 - .0546) \times \left(1 - 8 \times \left(\frac{.0546 + .0511}{2} - 0.0525\right)\right)\right] - 1 = 0.0446$$

Observations

Table 2 shows there would be no change in the liabilities to be funded in 12 out of 20 years. The Ontario PfAD approach, on the other hand, would require an increase in funding in 15 of the 20 years and a decrease in funding in the other five years.

The PfAD percentage would move up and down with long-term interest rates that fall within a specified range. It changes within the range of 1.96% to 17.95% over the 20-year period with a mean of 8.35%, which, incidentally, is close to the 8% prescribed in Ontario.

Comparing these statistics on percentage increase in liabilities with those under the Ontario PfAD approach –

Mean	2.18%
Standard deviation	3.15%
Maximum	8.98%
Minimum	0.00%

– it can be seen that the mean, standard deviation and maximum values under the alternative approach are all lower. This suggests that our proposed PfAD methodology would be more effective in stabilizing funding for pension plans in the face of interest rate changes.

4. British Columbia PfAD Design

The PfAD prescribed in BC is calculated as the greater of 5% or five times the long-term GoC bond yield (CANSIM series V122544), as long as the plan's non-fixed income allocation is 30% or more. Where the non-fixed income allocation is less than 30%, the PfAD is proportionately reduced but still subject to a floor of 5%. The PfAD determined under this formula is not dependent upon the investment policy or maturity of the plan. When interest rates are lower, liabilities are generally higher, but a lower PfAD is applied than when interest rates are higher. The funding target including PfAD is therefore expected to be more stable than that under the Ontario regulations in a changing-interest-rate environment.

Back-testing

We apply the PfAD methodology adopted in BC to the model plan over a 20-year period starting on January 1, 2000. Denote the long-term GoC bond yield at time t as y_t and the PfAD applied as $\overline{PfAD_t}$. The prescribed PfAD for the plan at time t can be written as follows:

$$\overline{PfAD_t} = \max(.05, 5y_t) = 5y_t,$$

since y_t is not less than 1% in the testing period.

Unlike the Ontario regulations, BC does not prescribe a BDR for defined benefit pension plans. For our analysis, we determine the "best-estimate" liabilities of the plan using the BDR prescribed in Ontario.

Consider two consecutive valuation dates, time 0 and time 1. Had there been no change to the GoC bond yield, the liabilities expected to be funded at time 1 would be equal to: $\left(1 + \overline{PfAD_0}\right) \cdot L_{1,0}$, where $L_{1,0}$ represents the liabilities determined at time 1 using a BDR of i_0. If the BDR at time 1 is changed to i_1 (due to a change in the GoC bond yield) and the PfAD is changed to $\overline{PfAD_1}$, the increase (decrease) in liabilities required to be funded at time 1 would be:

$$\Delta L_1 = \left(1 + \overline{PfAD_1}\right)L_{1,1} - \left(1 + \overline{PfAD_0}\right)L_{1,0} = \left(1 + \overline{PfAD_1}\right)(1 + \delta_1)L_{1,0} - \left(1 + \overline{PfAD_0}\right)L_{1,0}$$

where δ_1 is the adjustment factor applied to $L_{1,0}$ such that $L_{1,1} = (1 + \delta_1) \cdot L_{1,0}$, as defined in Section 2.

The increase (decrease) in liabilities required to be funded at time t, due to a change in the GoC bond yield, can be calculated as follows:

$$\Delta L_t = \left(1 + \overline{PfAD_t}\right)(1 + \delta_t)L_{t,t-1} - \left(1 + \overline{PfAD_{t-1}}\right)L_{t,t-1} \tag{6}$$

The last term in Equation (6) is the liabilities expected to be funded at time t if the GoC bond yield at that time is unchanged from time $t - 1$. The change in liabilities ΔL_t, expressed as a percentage of $\left(1 + \overline{PfAD_{t-1}}\right)L_{t,t-1}$, is therefore equal to: $\left[\frac{1+\overline{PfAD_t}}{1+\overline{PfAD_{t-1}}}\right] \cdot (1 + \delta_t) - 1$.

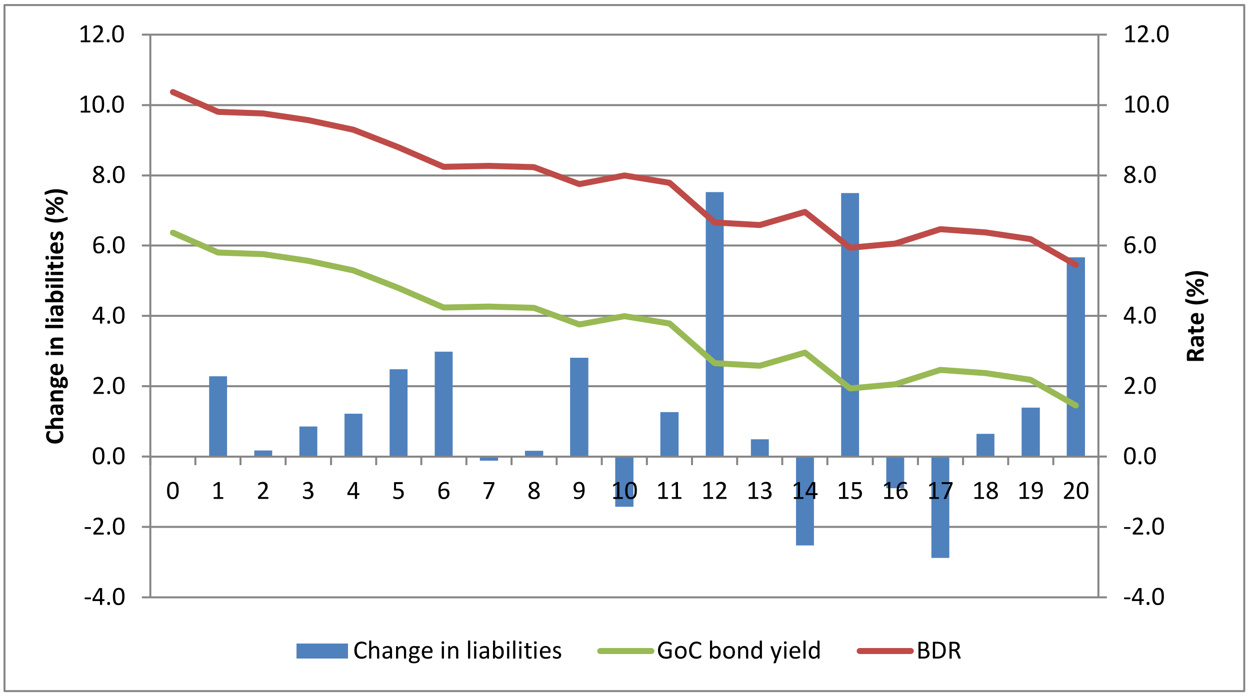
Table 3 shows the percentage increase (or decrease) in liabilities required to be funded, due solely to a change in the GoC bond yield, at each valuation date between January 1, 2001, and January 1, 2020 inclusive.

Table 3 – British Columbia PfAD Design

Valuation Date	Time	GoC Bond Yield (%)	BDR (%)	PfAD (%)	Change in Liabilities (%)
2000 Jan	0	6.37	10.37	31.84	
2001 Jan	1	5.80	9.80	29.01	2.28
2002 Jan	2	5.76	9.76	28.80	0.17
2003 Jan	3	5.57	9.57	27.83	0.85
2004 Jan	4	5.30	9.30	26.49	1.22
2005 Jan	5	4.80	8.80	23.98	2.48
2006 Jan	6	4.24	8.24	21.22	2.99
2007 Jan	7	4.26	8.26	21.32	(0.11)
2008 Jan	8	4.23	8.23	21.17	0.17
2009 Jan	9	3.75	7.75	18.77	2.81
2010 Jan	10	4.00	8.00	20.00	(1.43)
2011 Jan	11	3.79	7.79	18.93	1.26
2012 Jan	12	2.66	6.66	13.29	7.52
2013 Jan	13	2.59	6.59	12.93	0.49
2014 Jan	14	2.96	6.96	14.81	(2.52)
2015 Jan	15	1.94	5.94	9.70	7.50
2016 Jan	16	2.06	6.06	10.30	(0.90)
2017 Jan	17	2.47	6.47	12.33	(2.88)
2018 Jan	18	2.37	6.37	11.87	0.65
2019 Jan	19	2.18	6.18	10.91	1.40
2020 Jan	20	1.46	5.46	7.28	5.67

The PfAD percentage of 7.28% at January 1, 2020 is calculated as: $5y_{20} = 5 \times 1.455\% = 7.28\%$. The percentage increase in liabilities to be funded as at that date is 5.67%. This is calculated using the following formula: $\left(\frac{1+\overline{PfAD_{20}}}{1+\overline{PfAD_{19}}}\right)(1+\delta_{20}) - 1$, where $\overline{PfAD_{19}} = 0.1091, \overline{PfAD_{20}} = 0.0728$ and $\delta_{20} = 0.0924$ (obtained from Table 1).

The funding impact due to changes in GoC bond yields can be shown graphically as follows:



Observations

The PfAD percentage would be over 20% in the first decade of the testing period when long-term interest rates were in the 4% to 5% range. Plan sponsors might consider this level of funding margin to be excessive, especially if the plan is open and is pursuing a de-risking or liability-driven investment (LDI) strategy. The BC PfAD does not reflect a plan's investment risk exposure or maturity. This is a flaw in its design.

The average PfAD over the 20-year period is 18.7%, which is well above the PfAD of 8% under the Ontario rule.

The percentage increase in liabilities to be funded that is over 5% occurred at three valuation dates: January 1, 2012, 2015 and 2020. Among the other valuation dates, the maximum increase is no more than 3%. Below are relevant statistics over the 20-year period:

Mean	1.48%
Standard Deviation	2.84%
Maximum	7.52%
Minimum	(2.88%)

The mean, standard deviation and maximum values based on the BC PfAD formula are the lowest among the three design alternatives. This implies that the BC approach is effective in reducing long-term interest rate risk.

5. Conclusion

Our CIA Member Paper proposed a practical approach to establishing a funding margin that reflects a plan's investment policy, its level of maturity and the current level of long-term interest rates.

This paper presents a comparative analysis of three known PfAD design alternatives, regarding how effective they are in addressing interest rate risk. Our key findings are:

- The Ontario PfAD design is the least capable of stabilizing funding requirements in the face of long-term interest rate changes.
- The BC PfAD was designed to reduce long-term interest rate risk but it fails to reflect a plan's investment risk exposure and maturity. The PfAD derived from the BC formula could be considered excessive if the long-term interest rates move up to above 4% from the current historically low level.
- Our proposed PfAD methodology overcomes the shortcomings of the BC approach and has the potential to achieve stable funding for pension plans under a changing-interest-rate environment.

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