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EDUCATIONAL NOTE

Expected Mortality: Fully Underwritten Canadian Individual Life Insurance Policies

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Expected Mortality: Fully Underwritten Canadian Individual Life Insurance Policies

Committee on Life Insurance Financial Reporting

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Preamble

This educational note was originally developed in 2002 by the Committee on Life Insurance Financial Reporting (CLIFR) to assist in setting mortality assumptions for individual life insurance policies for Canadian Generally Accepted Accounting Principles (CGAAP) valuation.

Following the introduction of IFRS (International Financial Reporting Standard) 17, effective for fiscal years beginning on or after January 1, 2023, this document was updated in 2024 to remove specific references to IFRS 4 valuation. The scope of this review focused on required changes for IFRS 17 rather than trying to provide a comprehensive update.

However, we have taken the opportunity to (a) add references to newer research papers, (b) make environmental updates such as adding a COVID-19 section, (c) add clarity to the credibility formulas and numerical examples, and (d) remove the selective lapse appendix as we plan to merge the material into the guidance material covering selective lapsation.

Topics such as predictive modeling and its use in accelerated underwriting are not in scope for this educational note.

This educational note applies to Canadian individual life insurance business that is fully underwritten. However, many of the concepts covered will be useful to actuaries in establishing mortality assumptions for other types of business.

The educational note exposes general principles and processes applicable to determining an expected mortality assumption. It also provides a practical overview on how to apply credibility criteria and blend industry and company-/block-specific experience data to construct an expected mortality assumption.

Process

The creation of this educational note has followed the Actuarial Guidance Council's (AGC's) protocol for the adoption of educational notes. In accordance with the CIA's *Policy on Due Process for the Approval of Guidance Material Other Than Standards of Practice and Research Documents*, this educational note has been prepared by CLIFR and has received approval for distribution by the AGC on October 8, 2024.

Your feedback

Questions or comments regarding this educational note may be directed to the chair of CLIFR.

1. Introduction

1.1 Scope

This educational note restricts its guidance to Canadian individual life insurance business that is fully underwritten relating to the estimates of future cash flows under IFRS 17. However, many of the concepts covered will be useful to actuaries in establishing mortality assumptions for other types of business.

1.2 Definitions

When used in this educational note, "mortality table" means a table (or a set of tables) that reflect(s) the total mortality experience of a defined cohort of lives.[1]

"Simultaneous deaths" means the occurrence of a second death within six months of the first death and as a result of the same event.

[1] A mortality table can be constructed to reflect as many variables as are needed. For instance, the CIA2014 table is made up of tables covering gender and smoking status for standard underwritten Canadian industry business.

"Credible" means that the data are statistically reliable.

"Homogeneous" means of uniform structure or composition throughout.

1.3 General methodology

Methods used in deriving an expected mortality assumption for valuation would have the following characteristics:

a) The assumption derived is appropriate in aggregate for either the entire company or the particular block of business.

b) All relevant and material data are used, incorporating relevant component variation of mortality rates (e.g., sex, age, smoking status, amount, blood testing, duration).

c) The method results in an assumption that is unbiased.

d) The assumption derived is based on data that are homogeneous within each relevant class.

Where little credible experience is available, see subsection 6.1.

Where experience is 100% credible, companies may create mortality tables based on their own data.[2] The rest of this educational note would likely still be relevant in this situation.

The rest of this educational note focusses on assisting the actuary in setting the valuation expected mortality assumption when credible experience is available but it is not feasible to create a company table.

2. Assemble data

Relevant mortality data consist of company and inter-company experience studies. Additional sources of data might be assembled with these two types of studies in developing expected mortality assumptions.

2.1 Company experience

A company's own mortality experience for a particular block of business is usually the most relevant source of data. Often, company experience studies show mortality ratios for various time periods, and cross-sections of business, relative to an industry or internal table.

When a block of business is reinsured, the profile of the net retained business may be different than that of the gross or direct block. Therefore, it may be important to consider the characteristics of both the assumed and ceded blocks when setting the valuation assumption.

2.2 Inter-company experience

Inter-company experience studies examine large volumes of insurance business taken across the industry. An industry study provides credible results of insured population mortality. The disadvantage is that the distribution of business may not parallel that of the business block a company is valuing. Whenever possible, the actuary would choose studies that closely resemble the company's business. If none exist, other available data may guide the actuary.

Both the CIA and the Society of Actuaries (SOA) publish industry experience studies. Studies are also available on industry websites or in publications. (For example, websites list the SOA's large amount and older age studies; and actuarial organizations in the UK, Australia, and South Africa make mortality

[2] The mechanics of developing a mortality table based solely on company experience is outside the scope of this educational note.

studies available. Publications include the *North American Actuarial Journal*, and the *Product Development News*. Other mortality research is provided on a fee-for-service basis by private firms.)

Care would be taken in using non-adjusted mortality data from other countries in developing Canadian expected valuation mortality assumptions. There are differences in population mortality, underwriting standards, the socio-economic environment and product structures. The actuary would place less reliance on non-Canadian data, especially when there is uncertainty about a table's derivation or underlying data.

2.3 Other sources of data

Other sources of information (besides company or industry experience) may be considered if available. This is particularly true if business is new or unique, or there is insufficient credible experience data.

One source of data is government or private sector population studies examining large volumes of population experience. Insurance experience can differ substantially when considering the effects of underwriting, geographic location and choice of market. Care would be taken in the use of population studies. However, population studies can isolate trends in population mortality, cigar usage, etc. Population mortality studies can be used to fill the gaps when insured mortality studies are not available.

A second source of data is medical studies, which have previously been instrumental in developing finer mortality classifications (e.g., the smoker/non-smoker classifications). These studies can be useful in understanding how levels of underwriting affect mortality experience.

There are other studies done by private organizations, reinsurers or actuarial organizations that may be available.

3. Prepare data

3.1 Company experience

Data definition

The actuary would review available data and scrutinize its applicability to the business being valued. Studies would be reviewed, giving attention to data sources and handling, assumptions, and methodology for developing results. The last step of the review includes assessing adjustments to the data to better reflect the business being valued. Such adjustments might apply different weightings or adjust for mortality trends (improvement or deterioration) to update results to the valuation date. (See subsection 3.3 for further details.)

Other considerations:

a) Identify information in order to define homogenous experience blocks. If data are incomplete, pinpointing causes for experience changes becomes extremely difficult. For instance, if relaxed underwriting programs are not separated from normally underwritten policies, there will be problems identifying sources for subsequent deterioration in mortality.

b) Ensure relevant information is captured by the administration system. When considering new system designs, ensure essential information is captured. Older systems may not have all the required information. Since administrative practices may have invalidated some information, the actuary would check the existence of any "system workaround" procedures with the policy administration department. The actuary would need to understand what fields contain needed information for the study. Ideally, field coding would be sampled to confirm. Additionally, caution would be taken with any system conversions that create breaks in experience results.

c) Ensure administrative procedures use the system consistently. If the administration department does not clearly understand the appropriate transaction for situations, the information may be corrupt. These problems sometimes are undetected. Underwriting and cause of death data are especially susceptible to miscoding given that they tend to be more informational than transactional.

d) Document changes in underwriting methodology or class definitions as they occur. If changes are captured by the system, but not well understood at time of the experience study, insights may be lost.

Data validation

Review the extract specifications with knowledgeable systems people.

Summarize data, and validate them against other sources (e.g., Are death benefits paid consistent with financial statements? Is the mix of business by size, underwriting class, etc., consistent with sales statistics?).

Review study results for reasonableness against past studies, as well as intuitive tests (e.g., non-smokers are expected to have better mortality experience than smokers).

Where inconsistencies in the data can be clearly identified, the data would be adjusted. The problem blocks of experience would be excluded and considered separately from the study to remove any study bias if solutions to the inconsistencies are not evident and results would be materially affected.

3.2 Inter-company experience

Normally, users of inter-company experience do not directly validate data. Any user validation focuses on the applicability of the inter-company study results to the company.

In ensuring the data are appropriate, the actuary would review the methodology used in the study and consider weightings adjustments of inter-company experience to more closely match the distribution of the company business. Weighting by number of claims or by amount of claims is the usual practice.

3.3 Adjusting results to valuation date

The data that emerge from both industry and company experience studies are often several years old at the valuation date. Study results could be adjusted to reflect trends in mortality by applying an adjustment to a table from observed mortality experience to the valuation date. With respect to such updates, the actuary would make an assumption about improvement (or deterioration) between the date of the observed experience (usually the mid point of the experience study) and the valuation date.

If appropriate to the circumstance of the company, the actuary may use historical trends to extrapolate mortality improvement up to the valuation date. However, care would be taken using this methodology. The actuary would ensure that the data used in performing this analysis are homogeneous. Changes in business mix, or change in underwriting practices such as blood testing levels, could give a false perception of mortality improvement, where no improvement actually exists. For this reason, the actuary would use caution in trending overall mortality ratios from the recent Canadian industry experience.

The actuary would ensure that the data used to calculate mortality trends are credible. Reference to industry or population mortality trends may be appropriate even for large companies.

If homogeneous industry or company data cannot be obtained, the actuary may be able to get reasonable estimates of average mortality trends by reviewing population mortality. Population mortality tends to be based on a population of individual lives, whose composition changes slowly over time and therefore is reasonably homogeneous compared to insured data. However, even population data may be impacted by cohort effects and long-term society trends such reduction in smoker prevalence.

The actuary would consider whether apparent mortality improvement might have resulted from anomalies, such as COVID-19, or inconsistencies with the benchmark mortality table. (For example, as the business ages, and in the absence of any mortality improvement, a benchmark mortality table with an unjustifiably steep slope automatically produces mortality ratios that decrease over time.)

The actuary would consider adjustments for known events that may affect mortality trends but are not part of the trend data. An example of this is AIDS.

3.4 Other adjustments

The actuary might consider other adjustments to the data. Those include removing the anti-selective mortality present in the current experience and adjusting for any deficiencies in handling joint data in the construction of experience results (see subsection 6.3).

4. Determine differentiation

4.1 General considerations

The actuary would select factors for differentiating the mortality assumption (e.g., male/female, smoker/non-smoker). This selection process may be iterative as the desired differentiation may not be supported by available data (e.g., data are not split by the desired factor(s), or data post split are not sufficiently credible).

The actuary’s challenge is to determine predictive factors in differentiating mortality and choose a subset that balances credibility and accuracy. A key decision involves the number and identification of factors to use in differentiating the valuation mortality assumption.

To the extent that it makes a material difference to the estimates of future cash flows, the actuary would not make the same assumption for two policies unless they expect their experience to be similar.

Differentiation choices can materially alter estimates of future cash flows. In determining differentiation, the actuary would consider the following:

a) The credibility of the information (exercise caution in differentiation if estimates of future cash flows are sensitive, but credibility of the data supporting differentiation is low).

b) The actuary would be able to explain the connection between factors and mortality results.

c) The behaviour of differentiation over time (consider whether the effects wear off, remain level, or increase).

d) The correlation between factors (in crossing two or more factors, the possibility of double counting may lead to incorrect conclusions; for example, when the number of dimensions used in developing factors exceeds those for which raw mortality is available, the possibility of missing correlations between factors exists).

4.2 Potential factors

Current industry mortality tables differentiate mortality by at least four basic factors: age, sex, smoking status and duration. Large companies with sufficient experience to develop their own internal tables tend also to split their mortality assumptions by at least these four basic factors.

The annual CIA study of industry experience, relative to industry tables, can be used to assess how actual industry mortality has evolved, relative to the expected table over time. While mortality experience in aggregate may have improved since the construction of the table, the actuary would consider that the extent of improvement may differ among the four basic factors.

The actuary would also consider factors beyond these four basic factors, including but not necessarily limited to experience by face amount, type of underwriting, preferred risk classification and product type.

The annual CIA Mortality Study analyzes observed mortality by face amount band and is a good source of information. The actuary would carefully interpret study results to adjust for the impact of inflation and underwriting changes over time. However, since levels of underwriting are largely driven by age and face amount, the correlation with these factors cannot be overlooked (to avoid double counting).

Subsection 6.1 provides background on considerations concerning preferred risk classification. Currently, no comprehensive public Canadian industry study exists. Studies that estimate protective value of enhanced testing can be used to estimate impacts of more rigorous preferred testing.

The annual CIA experience study provides mortality ratios split by product type. Some differentiation can be found where average face amounts of the product types differ. In addition, other product-specific factors include those that impact policyholder anti-selection and overall lapse rates as well as the policyholder's purpose for insurance.

Depending on the circumstances, the actuary may consider differentiating by other factors such as distribution type and geography. Reinsurers may consider differentiating by ceding company.

5. Blend credible data

5.1 Overview

The available company and industry data, suitably prepared and segmented, can now be blended using credibility weightings. This section discusses criteria for a good credibility method and summarizes several credibility methods. Appendices 2 and 3 provide additional guidance and examples for selecting a method, as well as discussion of each method's advantages and disadvantages.

The goal of credibility theory is to provide a framework for combining data from different sets of observations. These may be prior and current observations, industry and company mortality rates, or other sets. For the purpose of this section, we will consider the two sets to consist of:

a) company data, which may not be fully credible; and

b) industry mortality tables or data, which are assumed to be fully credible.[3]

The actuary needs to select an assumption for expected mortality taking into account the sample information, as well as the underlying statistical distribution.

The normalized method is the preferred credibility method and 3,007 is the suggested number of deaths needed for full credibility.

[3] The basic assumption underlying the traditional use of credibility theory to set the valuation expected mortality assumption is that the industry mortality tables have 100% credibility. This assumption, however, may not hold if the actuary uses the industry data in its finer level of detail e.g., by gender, smoking status and year or duration. The actuary should review the amount of industry experience underlying the published data before attributing 100% credibility to the data.

5.2 Criteria for a good credibility method

The following are desirable characteristics of a good credibility method:

- practical to apply
- the sum of expected claims for the within-company subcategories is equal to the total company expected claims[4]
- all of the relevant information is used
- the results are reasonable in extreme or limiting cases
- the subcategory actual-to-expected mortality (A/E) ratios are reasonable relative to company and industry data (e.g., they fall within the range of corresponding industry and company experience A/E ratios)

5.3 Types of credibility theory

There are two major types of credibility theory: limited fluctuation credibility theory (LFCT), and greatest accuracy credibility theory (GACT).

While several approaches are discussed in this section and in Appendices 2 and 3, the normalized method, which is a type of LFCT, best meets the criteria for a good credibility method.

5.4 Limited fluctuation credibility theory[5]

LCFT provides a criterion for full credibility based on the size of the portfolio. Full credibility means it is appropriate to use only the portfolio's own experience and to ignore the entire industry data.

In addition, LFCT provides an ad hoc methodology for the determination of partial credibility, where there is a weighting of the portfolio's own experience and the industry experience.

The expected assumption for the aggregate amount of claims for a company for a year may be expressed as:

$$X_E = Z\bar{X} + (1 - Z)M$$

where

- X_E is the credibility-weighted expected aggregate amount of claims,
- Z is the credibility factor, or weighting given to sample data,
- $\bar{X}$ is the mean and is calculated from the company experience data $\mathbf{X}=\{X_1, X_2, \ldots, X_n\}$,
- M is the expected number or amount of death claims (or ratio of actual to expected death claims), based on the industry data for the same portfolio, which is assumed to be fully credible, and
- n is the number of observations.

In practice, the credibility factor is usually applied to the applicable A/E ratio, rather than to expected claims.

While this weighted average credibility formula has intuitive appeal, LFCT does not provide an underlying theoretical model for distribution of the X_i's, which is consistent with the formula.

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[4] Please refer to "Application of Limited Fluctuation Credibility Theory to subcategories of business" in subsection 5.4.

[5] Limited fluctuation credibility theory is also known as "American credibility."

In LFCT, one establishes whether $\bar{X}$ is fully credible by selecting a range parameter r (r > 0) and a probability level p (0<p<1) such that the difference between observed company experience $\bar{X}$ and its true underlying mean μ is small relative to μ.

The criterion can be written as

$$\Pr\{|\bar{X} - \mu| < r\mu\} \geq p$$

where r is the error margin, and p is the confidence level. Parameter values of p = 90% and r = 3% are interpreted as a 90% probability of being correct within a 3% margin of error.

Poisson model

Although the theoretical distribution for mortality is binomial, when the probabilities of the event (death, represented by the random variable X in the above formulas) are small, the Poisson distribution provides a reasonable approximation to a binomial distribution.

In the simple Poisson model, the only random variable is the number of claims, which is assumed to be Poisson.[6] Variations in claim size are ignored. If there is significant dispersion in the net amount at risk for each policy in the block under consideration, the use of a simple Poisson model may be inappropriate. The compound Poisson model incorporates the effect of variation in claim size and would normally result in a higher threshold of claims needed to reach the same credibility level. The compound Poisson model is discussed in Appendices 1 and 2.

Parameter values p = 90% and r = 5% are frequently cited as the minimum levels required for full credibility; however, there is no theoretical basis for determining these parameter values. When setting the expected mortality assumption for valuation purposes, one may want to use a higher threshold for full credibility, such as p = 90% and r = 3%. Prior to original publication in 2002, these parameters were the subject of many discussions within CLIFR. The consensus was that a minimum of 3,007 deaths would be recommended for 100% credibility. We expect that this issue will be revisited periodically as new literature and research emerges in this area. The actuary would justify the use of parameters p and r different than p = 90% and r = 3% for valuation purposes.

For p = 90% and r = 3%, the factor for partial credibility is defined by $Z = \min\left\{\sqrt{\frac{n}{3{,}007}}, 1\right\}$

Where n = number of claims in experience data and 3007 is taken from the standard normal table.

Number of claims	30	120	271	481	752	1083	1473	1924	2436	3007
Z	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

The parameters defined above are suggested for use in most situations. A significant dispersion in net amount at risk in the in-force block will increase volatility and could result in the need to use a higher number of deaths.

The use of the blending methodology set out in this section assumes that there exists relevant industry basis for blending. If there is no industry table or study that corresponds to the company's business mix, then it may be appropriate to assign a higher credibility factor to the company data than otherwise.

The number of claims needed for full credibility under other values of p and r are set out in the standard normal table in Appendix 2.

[6] See *Loss Models: From Data to Decisions,* Example 5.20 or *Introductory Credibility Theory,* Example 3.2.2.

The Poisson application can be extended to include data from more than one period or year. However, the number of years would be limited so that the mix and material risk characteristics of the portfolio are homogeneous over time.

Application of limited fluctuation credibility theory to subcategories of business

If the actuary wants to reflect experience split by subcategory (perhaps by sex, product or duration) but the experience in those subcategories is not 100% credible, the actuary would decide either to use the overall credibility factor or the lower credibility for the amount of experience in that subcategory. One can pool disparate distributions within the aggregate data under certain conditions.

If the relative proportions of the subcategories are stable over time, then an actuary can use the credibility factor based on the aggregate distribution of these heterogeneous Poisson distributions (i.e., the total company credibility factor) for each of the subcategories.

The requirement that the portfolio mix be stable with respect to the major subcategories over time may limit the applicability of this result. The portfolio mix may be regarded as stable over time if the proportions of the relevant subcategories are constant (both for the study period and the projection period for the future). If the study is based on smoking distinct business but new preferred underwriting classes are added to the portfolio, the assumption of stability of the portfolio mix may not hold.

If the relative proportions of the subgroups are not stable over time, and hence the assumptions do not hold, it may be appropriate to reflect the credibility of the subcategories in the determination of the expected mortality assumption. Assessing whether the conditions hold requires the actuary's judgment.

The normalized method is summarized below.

5.5 Normalized method – limited fluctuation credibility theory

The normalized method uses the credibility and the A/E ratios of the subcategories. However, the blended A/E ratios are adjusted to reproduce the expected claims level generated by the total company blended A/E ratio. The total expected claims for the company is the same as that produced using total company credibility, but the A/E ratios of the subcategories are used to allocate these deaths to the subcategories.

The normalized method has the following strengths:

- The sum of expected claims for the subcategories matches total expected claims, based on a blended A/E ratio, calculated at the total company level (i.e., the number of subcategories selected does not affect the overall result).
- All of the information is used: both total company and subcategory A/E ratios and credibility factors.
- The results are reasonable in extreme or limiting cases.
- The subcategory A/E ratios fall within the original range (or very close to the range).
- Interactive effects between subcategories may be captured.
- It is simple to apply in practice.

Normalized method

Step 1: Calculate the A/E ratios and credibility factors for the total company (or block) and for each of the subcategories.

Step 2: Calculate total company blended expected mortality ratio and corresponding expected claims using the total company credibility factor and total company mortality ratio from Step 1.

Step 3: Calculate total company blended expected mortality ratio and corresponding expected claims using the credibility factors and the A/E ratios from the subcategories.

Step 4: Adjust or “normalize” the A/E ratios of the subcategories by the ratio of the total expected claims from Step 2 to the total expected claims from Step 3.

Although this method does not have a strong theoretical base, it is pragmatic and satisfies the criteria for a good credibility method.

The following example sets out the normalized method using a simple Poisson model for claims. The compound Poisson model could also be used. Additional description of the Poisson and compound Poisson models are set out in Appendices 1 and 2.

Assume that a portfolio comprises six different subcategories: male and female; split in three groups (1, 2 and 3). For each subcategory, the distribution of number of claims is Poisson with a different parameter (for this purpose, assume that the random variable under consideration is the A/E ratio calculated for each category).

Step 1 is completed in the followed table:

Mortality experience data						
	Mortality ratios					
Industry data	**Male**	**Female**	**Total**			
Group 1	71.0%	75.0%	71.9%			
Group 2	84.0%	83.0%	83.8%			
Group 3	73.0%	85.0%	74.3%			
Total	**74.5%**	**78.7%**	**75.32%**			
	Mortality ratios			**Number of claims**		
Company data	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
Group 1	58.3%	45.7%	55.4%	63	15	78
Group 2	86.4%	93.2%	88.1%	44	15	59
Group 3	75.0%	105.9%	78.3%	54	9	63
Total	**69.7%**	**67.9%**	**69.3%**	**161**	**39**	**200**
	Company expected claims assuming industry mortality at 100 %			**Credibility factors with p = 90% and r = 3%**		
Company data	**Male**	**Female**	**Total**	**Male**	**Female**	**Total**
Group 1	108.1	32.8	140.9	0.14	0.07	0.16
Group 2	50.9	16.1	67.0	0.12	0.07	0.14
Group 3	72.0	8.5	80.5	0.13	0.05	0.14
Total	**231.0**	**57.4**	**288.4**	**0.23**	**0.11**	**0.26**

The credibility factor for the total company is 0.26 (calculated as $min\left\{\sqrt{\frac{200}{3,007}}, 1\right\}$), where 200 is the total number of claims for the company, and 3,007 is the factor from the normal table corresponding to p = 0.9 and r = 0.03.

Step 2: Calculate the total company blended expected mortality ratio and corresponding expected claims using the total company credibility factor and total company mortality ratio from the above table. The blended expected mortality ratio is 73.8% (calculated as 0.26 x 69.3% + 0.74 x 75.32%). The corresponding expected claims are 212.8 (calculated as 73.8% blended mortality ratio x 288.4 total expected claims using industry mortality table at 100%).

Step 3: Calculate the expected number of claims for the total company, using the claims and credibility of the subcategories.

	Blended expected mortality ratios – subcategory credibility			**Expected number of claims**		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Group 1	69.2%	72.9%	70.0%	74.8	23.9	98.7
Group 2	84.3%	83.7%	84.2%	42.9	13.5	56.4
Group 3	73.3%	86.1%	74.6%	52.8	7.3	60.1
Total	**73.8%**	**77.9%**	**74.6%**	**170.4**	**44.7**	**215.1**

Here, the ratio for each subcategory is the weighted average of the company and industry ratios; for example, the male group 1 ratio = 0.14 x 58.3% + (1-0.14) x 71.0% = 69.2%. Further, the total group 1 ratio is 70.0%, calculated as follows. First, the expected number of claims is calculated for each subcategory; for example, male group 1 = 69.2% x 108.1 = 74.8. Second, the total expected number of group 1 claims is the sum of the expected claims for the row = 74.8 + 23.9 = 98.7. Finally, the total group 1 ratio = 98.7 / 140.9 = 70.0%, where 140.9 is the expected number of group 1 claims at 100% of industry data.

At step 3 (i.e., before normalizing), the overall level of expected claims depends on the choice of subcategories. The greater the division of subcategories, the smaller the company credibility in each cell, and the closer results will be to industry experience. Thus, the choice of subcategories will affect the final subcategory mortality assumptions. Step 4: Normalize the A/E ratios from Step 3 by multiplying them by the ratio of the total expected claims from Step 2 to the total expected claims from Step 3.

The A/E ratios and corresponding expected number of claims by subcategory under the normalized method are set out in the following table:

	Blended expected mortality ratios – normalized method			**Expected number of claims**		
	Male	**Female**	**Total**	**Male**	**Female**	**Total**
Group 1	68.4%	72.1%	69.3%	73.9	23.7	97.6
Group 2	83.4%	82.8%	83.2%	42.4	13.3	55.8
Group 3	72.5%	85.2%	73.8%	52.2	7.2	59.4
Total	73.0%	77.0%	73.8%	168.5	44.2	**212.8**

Here the expected mortality ratio for group 1 male lives is 68.4%, which is equal to the blended ratio from Step 3 multiplied by the ratio of expected claims from Step 2 to that from Step 3 (calculated as 69.2% x 212.8 / 215.1 = 68.4%).

The normalized method allows for the calculation of credibility factors by subcategory but then produces the same number of expected claims in total for the company as if there was only one aggregate category.

The use of the blending methodology set out in this section assumes that there exists relevant industry basis for blending. If there is no industry table or study that corresponds to the company business mix, then it may be appropriate to assign a higher credibility factor to the company data than otherwise.

5.6 Buhlmann or greatest accuracy credibility theory

GACT or "European credibility" is based on work by Buhlmann. GACT has a better theoretical basis than LFCT and ensures that results are balanced, so normalization is obviated. GACT allows one to estimate within and between subcategory sources of variation.

GACT is theoretically complete and meets the criteria for a credibility method with one shortcoming. That shortcoming is that additional information about industry experience (beyond what is customarily collected and published) is required. Without these practical difficulties, GACT would likely be the preferred credibility method to use in determining the expected valuation mortality assumption.

There are several versions of GACT. The simpler Buhlmann model, and the slightly more complex Buhlmann-Straub model are outlined in Appendix 3.

5.7 Summary

From a theoretical point, the GACT method is preferable since it is theoretically complete. However, current industry data are not sufficiently detailed to support the use of GACT.

The normalized method, a variant of LFCT, is then the favoured approach. Despite the theoretical shortcomings, the normalized method meets all criteria for a good credibility method.

3,007 is the recommended number of deaths needed for full credibility. Dispersion in net amount at risk and the absence of credible industry data are two significant factors that would be considered when determining the number of deaths needed for full credibility.

5.8 Sources of information

For a more detailed explanation of the credibility theory, please refer to Appendices 1, 2, and 3, and the sources listed below:

- *Loss Models: From Data to Decisions* by Klugman, Willmot and Panjer, published by John Wiley and Sons
- "Introductory Credibility Theory" by Gordon E. Willmot, published by IIPR.
- "A Credibility Approach to Mortality Risk" by Mary R. Hardy and Harry H. Panjer, published by IIPR
- *Introduction to Credibility Theory* by Thomas N. Herzog
- 2019 SOA paper "*Credibility Methods Applied to Life, Health, and Pensions*" by David B. Atkinson
- 2019 CIA-SOA research paper: *The Application of Credibility Theory in the Canadian Life Insurance Industry*

6. Other adjustments

Following the steps outlined in Sections 2 through 5 result in a base mortality table. Other adjustments can now be made to reflect factors anticipated to influence the mortality experience. Some of the adjustments outlined below are specific to a particular segment of the business.

6.1 New underwriting techniques

Overview

Changes to underwriting techniques and testing thresholds are occurring all the time. Until sufficient credible insurance experience accumulates, the actuary would have to use estimates for the impact that these changes may have on mortality experience.

The actuary utilizes knowledge of mortality levels based on the old underwriting basis, together with the anticipated impacts of changes on the underwritten population, to derive the new mortality table. One method used to reflect improvements in underwriting involves the following formula:

$$Q(NEW) = Q(OLD) \times [1 - A - B - C \times (A + B)] / (1 - A - B)$$

Where Q(NEW), Q(OLD), A, B and C are defined as:

Q(NEW): The new mortality rate anticipated due to the underwriting changes.

Q(OLD): The old mortality rate, or current mortality, based on the old underwriting approach. If industry mortality has been used in developing these rates, the actuary would adjust for existing differences between the company and industry underwriting to avoid double counting.

A: The impairment frequency, or frequency that the underwriting technique will screen otherwise undetectable medical impairments. Medical laboratories often can estimate how frequently their tests detect impairments. If possible, the actuary would examine the company's own data on blocks where the test has been applied.

B: The sentinel frequency, or frequency that prospects with those impairments will avoid the company because of the underwriting change. This is difficult to estimate since by definition it represents a segment of the insured population that the company is not tracking. The risk is likely to occur whenever the prospective life insured is aware of a significant impairment, such as cocaine use, and has alternatives for obtaining insurance without screening. The nature and sophistication of the distribution system will also significantly impact this factor.

C: The additional mortality, or average amount of increased mortality that can be expected to occur in the impaired group defined by A and B. Estimates can often be obtained by discussion with the underwriting area and/or the medical director. The actuary would carefully examine the evidence supporting this assumption.

Application of this formula would consider the following:

- **Variations by age:** The mortality Q, the frequency of the impairment, and the average excess mortality represented by the impairment can be expected to vary by issue age. The calculation would be split into several age groups and interpolated.
- **Variations by duration since issue:** In the absence of reliable experience, the actuary could reasonably assume that mortality differentials would disappear over time with perhaps some residual differentials remaining.
- **Multiple underwriting technique changes:** If the actuary is reviewing more than one change in

underwriting technique at a time, care would be taken if the impairments uncovered by the techniques are not independent. Any correlation would be accounted for in setting the factor values.

- **Reliance on assumptions:** Many key assumptions of this formula are difficult to estimate with confidence. The actuary would consider the credibility of these assumptions, particularly if they have a material impact on policy liabilities.

Underwriting changes do not always improve mortality. In some cases, companies may remove a requirement to simplify the underwriting process or save costs. This formula can be utilized in reverse to address these situations.

Preferred underwriting / changes to underwriting classifications

The primary challenge in developing mortality assumptions for new underwriting classifications, such as "preferred" is the time needed for credible experience to develop. Industry experience studies may not exist and, even if they do, company differences in class criteria may jeopardize the applicability of results to any one company.

The lack of credible homogenous data does not diminish the importance of industry experience in evaluating company preferred mortality experience.

Even in the presence of credible early duration experience split by class, the actuary may still need to estimate the impact of the new underwriting classes on mortality over time. It is reasonable to assume that mortality rates for preferred and non-preferred risks would revert over time towards overall standard regular underwriting mortality rates.

In the absence of reliable and relevant experience, the actuary would review the length of time that effects of preferred underwriting persist. In these circumstances, it would be reasonable to assume that the effects of preferred underwriting wear off over the select period, that the effects of preferred underwriting wear off linearly between the last duration for which the insurer has reliable experience, and the duration at which the effects are expected to completely wear off.

In the absence of any credible and relevant mortality data for the preferred class, the actuary may use approaches similar to that outlined for new underwriting techniques. The actuary may calculate the mortality for the preferred class as if they were introducing tougher underwriting requirements or new underwriting techniques. Both situations involve estimating the impact of removing a segment of the insured population or class on the mortality of the remaining population or class. "B" in the formula below can be determined as [Q(OLD) – Q(NEW)] / Q(OLD).

A formula for splitting one class (say NS Standard) into two (NS Preferred, NS Residual) by applying tougher underwriting standards (e.g., blood pressure) is as follows:

Q(NS Preferred) = Q(NS Standard) x (1 – B)

Q(NS Residual) = Q(NS Standard) x (1 – A + B x A) / (1 – A)

Where Q(NS Preferred), Q(NS Standard), A and B are defined as:

Q(NS Preferred): The preferred mortality rate, or mortality anticipated for applicants who qualify under the tougher underwriting standard.

Q(NS Standard): The standard mortality rate, or mortality currently experienced for the aggregate standard class undifferentiated by the underwriting criteria.

A: The preferred proportion, or frequency that a normally standard client will be accepted for preferred classification due to the tightened requirements. This can be a difficult task since most companies often do not store lab test results for future analysis. Medical directors, lab testing companies and reinsurers can be valuable resources in determining estimates. If available, the actuary would examine the company's own data.

B: the mortality differential for the preferred class relative to the old standard class. Estimates for this figure can often be obtained by discussions with the underwriting area and/or the medical director. The actuary would carefully examine the evidence supporting this assumption. The preferred underwriting class will normally be defined by more than one underwriting criterion. The estimates for A and B can be developed with the aggregate impacts of the various criteria in mind. The actuary would reflect any correlation between the health criteria in the determination of these assumptions. If the underwriting criteria are independent, qualifying percentages and mortality ratios can be developed for each independently and then multiplied to obtain the final result. A thorough review of underwriting files can assist the development of these assumptions.

When dealing with multiple underwriting classes, repeat the procedure as many times as required. Start with the class having the strictest underwriting requirements, and successively refine each class in each further step.

If a new underwriting technique is added at the same time as new classifications are developed, the change in overall mortality due to the new underwriting technique would first be quantified, before examining the relationship of preferred to standard mortality.

There are various considerations for applying this formula in practice:

- **Variations by age:** The additional mortality and qualifying percentages can be expected to vary by issue age, so the calculation would be split into several age groups and interpolated.
- **Variations by duration:** It is reasonable to assume that effects of preferred underwriting wear off over the select period.
- **Reverse sentinel effect:** Competitors' criteria for preferred classes may differ, so one company may lose the best risks of its classes to its competitors, and another may gain. The net result may concentrate poorer risks in each of its classes, but it is difficult to estimate. The importance of examining actual to expected mortality experience as credible experience develops becomes even more important if a company's underwriting classes differ considerably from the industry.
- **Reliance on assumptions:** Key assumptions are difficult to estimate with confidence. The actuary would consider how much confidence they have in the assumptions, particularly if these assumptions materially impact estimates of future cash flows. Where confidence does not exist, the actuary could value all new risk classifications that comprise the original standard classification using one aggregate mortality assumption, and consider a higher risk adjustment for non-financial risk.
- **Reinsurance:** Care would be taken in using the reinsurer's premium rates as a proxy for the valuation expected mortality assumption. Despite the insurer substituting their mortality cost for a fixed rate basis, an independent assessment of the underlying mortality may be required. A reinsurer's rates are usually simple multiples of a standard industry table, for ease of sale and comparison. Actual experience on new risk classifications will likely vary by age and duration.

Example: Suppose a company is simultaneously introducing tougher underwriting testing and splitting standard non-smoker classes into two, based on a set of underwriting criteria. Suppose further:

- The condition detected by the new underwriting tests is present in 2% of insurance applicants on average.

- The company is late relative to industry in adding this test to its requirements.
- It is estimated that there are an additional 1% of applicants with the condition that seek insurance because of the company's weaker underwriting regime.
- The mortality experience used as a base reflects the additional mortality costs associated with this discontinuity.
- The mortality of these applicants is 500% of the normal standard underwritten case (so the additional mortality is 400%).
- The current duration 1 mortality rate assumed for a female non-smoker aged 60 is $1/1000.

To determine the new aggregate mortality rate once the test is implemented:

Qnew = $1/1000 x (1 – 0.02 - 0.01 - (0.02 + 0.01) x 400%) / (1 - 0.02 - 0.01)

= $1/1000 x 0 .85 / 0.97 = $0.88/1000

Next, assume the company splits the new non-smoker class into two, based on a set of qualifying criteria they assume will separate risks into the top 40% and residual 60%. The preferred risks are anticipated to have 15% lower mortality than anticipated in the otherwise aggregate class. In this case, the preferred and residual class mortality can be calculated as:

Qnew (preferred) = $0.88/1000 x (1 - 0.15) = $0.748/1000

Qnew (residual) = $0.88/1000 x (1 - 0.4 + 0.15 x 0.4) / (1 - 0.4) = $0.968/1000

As a check on the results, the actuary can perform a test to show that the new assumptions produce the same overall mortality as the old assumptions.

[Qnew (preferred)] x 0.4 + [Qnew (residual)] x (1 - 0.4) = 0.748/1000 x 0.4 + 0.968/1000 x 0.6 = $0.88/1000

6.2 Selective lapsation

Selective lapses are defined as lapses whose mortality experience would be identical to that of newly selected lives.

The actuary would consider effects of selective lapsation when setting the expected mortality assumption even though selective lapsation is difficult to observe (as the health of lapsed policyholders is unknown). Selective lapsation is typically modelled as an explicit adjustment to the base/expected mortality. Refer to the educational note supplement: *Selective Lapsation for Renewable Term Insurance Products* for more information.

6.3 Multiple life policies

The actuary would calculate expected mortality using multiple life contingencies, with exact ages, sex, smoking status, and substandard rating, whenever possible. However, determining the mortality assumption for policies with multiple lives may not be possible in all situations. For example, the required data for each life may not be available, especially for older business. For these reasons, the actuary may consider using approximations.

Approximation methods

Equivalent single age (ESA) and joint equal age (JEA) are two approximation approaches used in situations where the exact method is not possible. Unfortunately, both may generate a mortality curve significantly different from actual joint mortality calculated from first principles. The approximations in these cases would at best be appropriate only for a short period and diverge from the exact result over time. The approximations can also cause period-by-period differences between actual and expected mortality which makes emerging experience more challenging to analyze.

Equivalent single age

Under the ESA approach, the joint mortality is approximated using the mortality of a single age that would have roughly the same present value of death benefits. A set of rules is defined for converting from the actual ages of the joint lives to an ESA.

Single life mortality has a very different slope than joint mortality. For joint last survivor (JLS), the mortality rates under an ESA approach are significantly higher in early durations than exact mortality developed via first principles. At later durations, the relationship reverses, and JLS mortality is higher than the ESA mortality. An ESA calculated at issue will understate the estimates of future cash flows beyond the first duration.

The actuary could improve the approximation by recalculating the ESA at each valuation date, but this requires complete knowledge of each life making up the ESA, which may not be practical. Alternatively, the actuary could estimate a set of factors for each future valuation date to apply to the ESA estimate of future cash flows to produce more accurate estimates of future cash flows for joint policies by first examining the ratios of the joint estimates to the ESA estimates for various ages, sexes and smoking statuses.

The following example helps to illustrate the size of this difference:

Policy type:	JLS
Life 1:	Male NS, age 45
Life 2:	Female NS, age 40
Mortality:	86.5% 86-92 CIA, age nearest birthday
Interest rate:	6%
ESA:	Male NS, age 30

	Present value of future death benefits		
Duration	Joint	ESA	Difference
0	0.0671	0.0676	0.0005
20	0.2130	0.1881	-0.0249
40	0.5620	0.4573	-0.1047

The opposite relationship occurs with joint first to die (JFS) mortality. Early duration ESA mortality rates are lower than actual mortality rates, calculated from first principles, while later duration mortality rates are higher.

Joint equal age

Under the JEA approach, mortality rates are approximated using joint mortality of the same number of multiple lives, with a single age and underwriting class. The JEA is selected to approximate the same present value of death benefits. Rules are defined for converting from actual ages to the JEA.

The JEA approach is superior to the ESA since the mortality slope better matches an exact age approach. However, the actuary would ensure the present value of future mortality using the approximation that is still appropriate.

Mortality studies involving joint lives

Mortality studies on joint life business are rarely credible. The actuary would ensure that any mortality studies performed on joint life policies are conducted, and interpreted, correctly. The following issues would be considered:

- **First death reporting:** The most accurate method to study mortality on multiple life policies is to compare deaths on each life. This is relatively easy on JFS policies, since the reporting of deaths is the same as for single lives. However, this approach may be impractical for JLS policies if material numbers of first deaths are not reported.
- **Choice of expected mortality:** The choice of expected mortality for a study of multiple life policies presents unique challenges. The lack of any multiple life industry studies necessitates the use of single life mortality tables. The actuary would ensure that the table selection is appropriate. For example, the actuary might choose the expected mortality for single lives that most closely matches the average underwriting characteristics for multiple lives, as multiple life policies may be larger on average.
- **Incidence of substandard lives:** Since a significant number of JLS policies are issued with one life substandard, JLS policies generally have a higher incidence of substandard lives than the single life portfolio. Some companies adjust ESAs rather than apply a rating, which may make tracking substandard experience difficult.
- **Credibility:** Refining data into credible subgroups is more difficult for JLS than for single life business. The early duration credibility for JLS business is significantly lower than a similarly sized block of single life policies due to the low probability of claim. So, larger in-force blocks are needed relative to single life policies. In addition, the number of policy combinations grows exponentially.
- **Use of approximations:** The actuary would exercise caution when using an expected table developed using the ESA or the JEA method. For example, the expected basis for a JLS block of business calculated using an ESA approach will show very favourable early duration expected claim experience. However, this expected claim experience will deteriorate for later durations.
- **Application of mortality improvements:** The actuary would use caution in any application of single life mortality improvement factors to JLS claim experience.

Simultaneous deaths on joint last survivor policies

For two totally unrelated lives with no regular interaction in their day to day lives, the probability of simultaneous death is remote. However, people who have a reason to buy a JLS life insurance policy will often have regular interactions increasing the risk of simultaneous death. If this risk is not considered, the mortality assumption may be understated.

Suggested readings

The following is a list of articles published by the SOA on the topic of multiple life mortality[7]:

- *The Actuary*, January 1994, by Jack Bragg, Jack Luff and Bob Vose
- Last Survivor Insurance Antiselection – SOA Product Development News, February 1994, Craig W. Reynolds;

[7] This list has not been updated following the original version of this educational note in 2002.

- “Second-to-Die with Possibility of Simultaneous Death” – SOA Product Development News, June 1994, Harry H. Panjer

6.4 Other Significant Events

Other significant events may cause expected future mortality to be materially different from historical experience and require special adjustments. When the event first occurs, it is likely that theoretical approaches will be used to develop such adjustments because experience is not available.

As experience emerges, it becomes important to estimate how much of the expected mortality impacts are already reflected in the experience, and what adjustments are required before using that experience to estimate future mortality rates. The adjustments may be to increase or decrease mortality rates, depending on whether future mortality rates are expected to be higher or lower than historical observed experience.

Adjustments may be needed to both the base level of mortality and mortality trends.

Two examples of other significant events are AIDS and COVID-19.

6.4.1 AIDS

When the AIDS epidemic first emerged, there was no data on the effect of AIDS on insured life mortality. The CIA promulgated a general theoretical methodology for reflecting the level of AIDS mortality in the estimates of future cash flows for individual life insurance. This general methodology used an AIDS model based on population mortality.

The population mortality was to be adjusted to represent insured mortality using a number of factors. These factors are set out later in this section.

As experience emerged, it became important to recognize the degree to which AIDS mortality is already included in the experience data. An explicit AIDS provision was no longer required if the actuary considers that AIDS claims are fully included in the experience. When determining the extent to which AIDS is included in the experience, the actuary would consider the following:

- AIDS claims as a percent of total claims for own company experience relative to comparable industry experience or population experience;
- the degree to which AIDS deaths are reflected in experience may vary by issue date and issue age;
- target markets; and
- historical underwriting standards and testing limits.

In addition, the actuary would consider medical changes with respect to the treatment of AIDS and the impact that these changes will likely have on mortality experience.

To the extent that the actuary believes that AIDS is not fully included in the experience data, the actuary would adjust the expected mortality, taking into account the following factors:

- **Assumed Level of Ownership:** A lower portion of the at-risk group than the overall population will own individual life insurance. The minimum recommended assumed proportion for pre-1984 extra AIDS mortality was 40%. This proportion would be updated to the current valuation year assuming selective lapsation.

- **Assumed parameters of the AIDS Epidemic:** for example, the pattern of future infections, the incubation time, the development of clinical AIDS to death, and the distribution of AIDS cases in the population by age.
- **Effect of HIV testing:** At many life insurance companies, specific testing for HIV was introduced in the late 1980's or early 1990's. For some companies, the threshold for testing was reduced at a later date. These testing levels may be important to consider when reviewing company experience.
- **Effect of Selective Lapsation:** It is reasonable to assume that people who know they have AIDS will be unlikely to surrender their policies. This assumption may also apply for those who are HIV positive and to a lesser degree for those in a high-risk group. The selective lapsation methodology set out elsewhere in this note could be applied.
- **Geographic Differences:** The incidence of AIDS may vary by territory and within territory by region.
- **Company Characteristics:** Different companies may have different AIDS experience, depending upon the target market (urban versus rural), age and sex distribution, and underwriting limits.

6.4.2 COVID-19

The COVID-19 outbreak was declared a pandemic by the World Health Organization (WHO) in March 2020. In May 2023, the WHO announced that, while COVID-19 continues to be a global health threat, the pandemic no longer qualifies as a global emergency. Nevertheless, the challenges facing actuaries in assumption setting continue to evolve.

One area of interest and importance to many actuaries is the impact of COVID-19. Beginning in October 2020, the CIA and representatives of several Canadian life insurance companies published six research reports assessing COVID-19's impact on the industry. The October 2022 report was the final one in the series.

A main challenge area relates to experience studies. The actuary may need to assess how to best incorporate the 2020 to 2022 exposure years into the experience studies and assumption setting including historical mortality improvement. This decision may at least partially be based on the actuary's judgment on the long-term impact of the COVID-19 pandemic. The long-term effects of COVID are still largely unknown but could impact future mortality experience.

Appendix 1 – Probability and statistical concepts

Probability concepts

Poisson distribution

The number of claims of a portfolio of business may be described as a Poisson model.

If X and Y are independent Poisson random variables with Poisson parameters a and b respectively, then

E[X] = Var[X] = a,

E[Y] = Var[Y] = b, and

W = X + Y is also Poisson with parameter c = a + b (and E[W] = Var[W] = c = a + b).

We will refer to such a distribution as an aggregate Poisson distribution.

Also, if we know W is a Poisson random variable with parameter c, then we know that W may be decomposed into 2 or more Poisson random variables with respective Poisson parameters summing to c.

These are the addition and decomposition properties of the Poisson model.

Although the theoretical distribution for mortality is binomial, when the probabilities of the event (death) are small, Poisson is a reasonable approximation to binomial.

Compound Poisson distribution

The total claims of a portfolio of business can be described as a compound Poisson model, which reflects both the number and amount of claims. When the additional consideration is given to the variability of claims sizes, the threshold for full credibility is increased relative to the Poisson model.

Let N be a random variable representing the number of claims from an insurance company and assume that N has a Poisson distribution with mean and variance λ. The observed number of claims is n.

For k = 1, 2, 3,…n, let Y_k be the random variable representing the amount of the k^{th} claim.[8]

Assume that Y_k's are independent and have a distribution with mean μ_y and variance $\sigma_y{}^2$.

Assume that the number of claims N is independent of the claim size Y_k.

The total claims $X = Y_1 + Y_2 + Y_3 + \ldots + Y_N$ has a compound Poisson distribution.

Using conditional expectation on N, it can be shown that

$E[X] = \mu = \lambda\mu_y$, and

$Var[X] = \sigma^2 = \lambda(\mu_y{}^2 + \sigma_y{}^2)$

In summary, X_i has a compound Poisson distribution with Poisson parameter λ and claims size distribution with mean μ and variance $\sigma_y{}^2$.

Estimators

Consider a portfolio of n life insurance policies numbered 1, 2, 3, …, n with corresponding one-year mortality rates $q_1, q_2, q_3, \ldots, q_n$ and corresponding net-amounts-at-risk $b_1, b_2, b_3, \ldots, b_n$.

[8] The random claim size comes in this way: Given that a claim from the portfolio occurs, what are the probabilities of the various possible amounts that this claim can be? The answer is: For any amount, it is the sum of the mortality rates for all.

For a one-year period, the mean and standard deviation of the total number of deaths and aggregate death claims are:

	Numbers of deaths	Aggregate death claims
Expected:	$\lambda = \sum_{i=1}^{n} q_i$	$\mu = \sum_{i=1}^{n} q \cdot b_i$
Standard deviation:	$\sqrt{\sum_{i=1}^{n} q_i(1-q_i)}$	$\sqrt{\sum_{i=1}^{n} q_i(1-q_i)b_i^2}$

For large values of n, the distribution of the numbers of claims and the aggregated claims are approximately normally distributed with the above means and standard deviations.

The Poisson distribution with mean λ can also be used as an approximation to the number of deaths. The standard deviation of the Poisson distribution is $\sqrt{\lambda}$ which is slightly larger than the true standard deviation given above.

The corresponding compound Poisson distribution can be used to approximate the distribution of aggregate claims. It has mean μ and standard deviation $\sqrt{\sum_{i=1}^{n} q_i b_i^2}$ which is slightly larger than the true standard deviation given above.

<u>Statistical concepts</u>

Summary statistics

Define the mean amount of aggregate claims per year as

$$\bar{X} = \frac{\sum_{i=1}^{m} X_i}{m}$$

where m represents the number of years of experience for the company.

Then[9]

$\mathrm{E}[\bar{X}] = \mu = \lambda\mu_y$ and $\mathrm{Var}[\bar{X}] = \sigma^2 = \lambda(\mu_y{}^2 + \sigma_y{}^2)/m$

Central limit theorem

According to the central limit theorem, if the amount of experience is "large," then the random variable

$$\frac{(\bar{X} - x)}{\sqrt{\mathrm{Var}(\bar{X})}}$$

is approximately distributed as a normal random variable with mean zero and standard deviation one (*x* is the true value of X).

[9] See *Introductory Credibility Theory,* Example 2.2.3.

Appendix 2 – Limited fluctuation credibility theory[10]

LFCT provides a criterion for full credibility based on the size of the portfolio. Full credibility means it may be appropriate to use only the portfolio's own experience and to ignore the industry data.

In addition, LFCT provides an ad hoc methodology for the determination of partial credibility, where there is a weighting of the portfolio's own experience and the industry experience.

The expected assumption for the aggregate amount of claims for a company for a year may be expressed as

$$X_E = Z\bar{X} + (1 - Z)M$$

where

- X_E is the credibility-weighted expected aggregate amount of claims,
- Z is the credibility factor, or weighting given to experience data,
- $\bar{X}$ is the observed mean and is calculated from the experience data **X**=$\{X_1, X_2, \ldots, X_n\}$,
- M is the expected number or amount of death claims, based on the industry data for the same portfolio, which is assumed to be fully credible, and
- n is the number of observations.

While this weighted average credibility formula has intuitive appeal, LFCT does not provide an underlying theoretical model for distribution of the X_i's, which is consistent with the formula.

In LFCT, one establishes whether $\bar{X}$ is fully credible by selecting a range parameter r (r > 0) and a probability level p (0<p<1) such that the difference between $\bar{X}$ and its mean μ (i.e., the observed company experience compared to its true underlying mean) is small relative to the true underlying mean.

The criterion can be written as

$$\Pr\{|\bar{X} - \mu| < r\mu\} \geq p$$

where r is the error margin, a "small" number and p is the confidence interval, a "big" number.

Parameter values p = 90% and r = 5% are frequently cited as the minimum levels required for full credibility; however, there is no theoretical basis for determining these parameter values. When setting the expected mortality assumption for valuation purposes, one may want to use a higher threshold for full credibility, such as p = 90% and r = 3%.

In many situations, it is reasonable to approximate the distribution of $\bar{X}$ by a normal distribution. There are various models for the underlying distribution of claims. Poisson and compound Poisson are discussed here.

Poisson model

Although the theoretical distribution for mortality is binomial, when the probabilities of the event (death) are small, the Poisson distribution provides a reasonable approximation to a binomial distribution.

[10] American credibility.

In the simple Poisson model, the only random variable is the number of claims, which is assumed to be Poisson.[11] Variations in claim size are ignored. The following table sets out the number of claims needed for full credibility for various values of p and r.

Standard normal table – range and probability parameters					
Number of claims needed for full credibility					
Probability parameter p	**Range parameter r**				
	5%	**4%**	**3%**	**2%**	**1%**
90%	1,082	1,691	3,007	6,765	27,060
95%	1,537	2,401	4,268	9,604	38,416
99%	2,654	4,147	7,373	16,589	66,538
99.9%	4,331	6,767	12,030	27,068	108,274

For p = 90% and r = 3%, the factor for partial credibility is defined by $\mathrm{Z} = \min\left\{\sqrt{\frac{n}{3{,}007}}, 1\right\}$ where n = number of claims in experience data.

Number of claims	30	120	271	481	752	1083	1473	1924	2436	3007
Z	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

In other words, for p = 90% and r = 3%, one gets full credibility if the number of claims in the exposure period is greater than or equal to 3,007. The credibility formula can be viewed as the square root of the ratio of the number of claims in the data to the number of claims needed for full credibility. This is derived by requiring that the variance of X_E to be sufficiently small.

Below is an example of the application of the Poisson model.

Example 1	**Industry data**	**Company data**
Source of data	From industry mortality study	From company study for same period
Mortality ratio	75.3%	69.4%
Observed number of claims	Not needed	200
Credibility factor	1.00	$(200/3{,}007)^{.5} = 0.26$
Blended A/E claim ratio		0.26 x 69.4% + 0.74 x 75.3% = 73.8%

The Poisson application can be extended to include data from more than one period or year. However, the number of years would be limited so that the mix and material risk characteristics of the portfolio are homogeneous over time.

[11] See *Loss Models: From Data to Decisions,* Example 5.20 or *Introductory Credibility Theory,* Example 3.2.2. Or, *Loss Models: From Data to Decisions 3rd Edition,* subsection 20.2.2.

Compound Poisson model

Using the compound Poisson model with r = 3% and p = 90%, it may be shown[12] the number of deaths needed for full credibility is given by

$$C = \left\{ 3007 \times \frac{\left(\sum_{i=1}^{n} q_b b_i^2\right)}{\left(\sum_{i=1}^{n} q_i b_i\right)^2} \frac{\left(\sum_{i=1}^{n} q_i\right)}{\square} \right\}$$

where b_i =net amount at risk for policy i

q_i =one-year mortality rate for policy i

i = 1, 2, 3, …, n

In order to calculate Z for compound Poisson, one needs to calculate mean μ_Y and standard deviation σ_Y of the claim size distribution. These may be calculated from the total exposure or may be estimated using the actual claims.

For full credibility, the number of deaths in the portfolio's own experience must exceed this number C.

When the criterion for full credibility is not met, partial credibility can be applied. For p = 90% and r = 3%, the square root rule gives the credibility factor Z as

$$\mathrm{Z} = \min\left\{ \sqrt{\frac{X}{C}}, 1 \right\}$$

where C is the criterion for full credibility and X is the observed number of deaths in the experience of the portfolio. If only number of deaths are considered, C = 3007.

The compound Poisson example can be extended to include data from more than one period or year, where

N_j is a random variable representing the number of claims during period j, j = 1, 2, 3, …, m.

$Y_{j,k}$ is the random variable representing the amount of the k^{th} claim in period j.

X_j is the random variable denoting the aggregate claims amount for the company in period j and is defined by $X_j = Y_{j,1} + Y_{j,2} + \ldots + Y_{j,n}$.

However, the number of years would be limited so that the portfolio is homogeneous over time.

The details of the derivations of all of the formulas are given in *Loss Models: from Data to Decisions*, by Klugman, Panjer and Willmot, published in 1998 by John Wiley and Sons.

Below is an example of the application of the compound Poisson Model.

[12] Appendix 1, Estimators.

Example 2	Industry data	Company data
Source of data	From industry mortality study one year	From company study for same period
Mortality ratio	75.3%	69.4%
Number of claims N	Not needed	200
Claim rate q_i		0.001
Claim size b_i		50 at 50,000 50 at 100,000 50 at 150,000 50 at 200,000
Credibility factor	1.00	0.24
Blended A/E claim ratio		0.24 x 69.4% + 0.76 x 75.3% = 73.9%

The key assumptions underlying the compound Poisson formula for credibility are:

- The distribution of claim amounts Y_k are independent and have a distribution with mean μ_y and variance σ_y^2.
- The number of claims N is independent of the claim size Y_k.
- Central limit theorem is used to approximate the distribution of $\left((X_i - x)/\sigma_x\right)$ as a normal random variable with mean zero and standard deviation one.[13]

[13] See page 193 of Hogg and Craig (1978).

Appendix 3 – Greatest accuracy credibility theory/Buhlmann model[14]

Overview

The GACT allows one to estimate within and between subcategory sources of variation.

GACT is theoretically complete and meets the criteria for a credibility method with one shortcoming. That shortcoming is that additional information about industry experience (beyond what is customarily collected and published) is required. Without these practical difficulties, GACT would likely be the preferred credibility method to use in determining the expected valuation mortality assumption.

There are several versions of GACT. One of the simplest, the Buhlmann model, is discussed here. A more complex model, the Buhlmann-Straub is outlined in latter sections of this appendix.

Buhlmann model[15]

Assume for a particular policyholder or risk class, we know the past claim experience X={X_1, X_2,..., X_n } and that it is distributed with the same mean and variance, conditional on θ. For our purpose, assume that X is the experience of a particular company. The industry data comprises experience from many companies.

The policyholder has been categorized by underwriting characteristics and we have a "manual" rate that reflects these underwriting characteristics. The rating class is viewed as homogeneous with respect to the underwriting characteristics, but even within this rating class there is some heterogeneity (good risks and bad risks) since no rating classification can be detailed enough to capture all information.

Assume that this residual variation in the risk level of each policyholder in the portfolio may be characterized by a parameter θ (possibly a vector), but that θ for a given policyholder cannot be known.

Assume further that the cumulative distribution function $\mathrm{B}(\theta) = \Pr\{\Theta \leq \theta\}$ is known. $\mathrm{B}(\theta)$ represents the probability that a policyholder picked at random from the rating class has a risk parameter less than or equal to θ.

Assume that the claims experience of a policyholder can be expressed as the following conditional distribution:[16]

$$f_x \mid \Theta\big(x_j \mid \theta\big\}, \mathrm{j} = 1,2, \dots, \mathrm{n}, \mathrm{n}+1$$

Assume that the past claims experience X = {X_1, X_2, …, X_n} is distributed with the same mean and variance, conditional on a risk parameter which is not known for a particular policyholder.

Define

$\mathrm{u}(\theta) = \mathrm{E}\big(X_j \mid \Theta = \theta\big)$ (hypothetical mean)

$\mathrm{v}(\theta) = \mathrm{Var}\big(X_j \mid \Theta = \theta\big)$ (hypothetical variance)

$\mathrm{u} = \mathrm{E}\{\mathrm{u}(\theta)\}$ (pure premium)

$\mathrm{v} = \mathrm{E}\{\mathrm{v}(\theta)\}$ (expected value of process variance [or variability within company])

$\mathrm{a} = \mathrm{Var}\{\mathrm{u}(\theta)\}$ (variance of hypothetical means [or variability between companies])

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[14] European credibility

[15] *Loss Models: From Data to Decisions*, subsection 5.4.3, or Introductory Credibility Theory, Section 4.3.

[16] *Loss Models: From Data to Decisions*, subsection 5.4, and Chapter 4 of Introductory Credibility Theory.

It can be shown that the credibility factor is of the form:

$$Z = \frac{n}{n+k}$$

where

$$k = \frac{\text{expected value of process variance}}{\text{variance of hypothetical mean}} = \frac{v}{a}$$

As *a* (the variance of means across companies) decreases, *k* increases, and credibility factor Z decreases (if there is little difference between companies, more weight would be given to the industry experience, which will be less subject to random fluctuation).

As *v* (the expected value of the variability within the company) decreases, *k* decreases, and Z increases (if there is little fluctuation within the company, its own experience is more representative of the expected future experience).

For example, if $\{X_j \mid \Theta; j = 1,2,3, \dots, n]$ are independently and identically Poisson with given mean Θ, and Θ is Gamma with parameters a and b, then

$Z = \frac{n}{n+1/b}$[17]

The amounts *v* and *a* may be estimated using non-parametric estimators of the form[18].

$$\hat{v}_i = \frac{1}{n-1}\sum_{j=1}^{n} \left(X_{ij} - \bar{X}_i\right)^2 \qquad \hat{v} = \frac{\sum_{i=1}^{r} \sum_{j=1}^{n} \left(X_{ij} - \bar{X}_i\right)^2}{r(n-1)} \qquad \hat{a} = \frac{1}{(r-1)}\sum_{i=1}^{r} (\bar{X}_i - \bar{X})^2 - \frac{\hat{v}}{n}$$

Example #3 uses parametric estimators described in more detail in Appendix 2.

Example 3 – Buhlmann

A/E ratios by year			
Experience study year	**Company 1**	**Company 2**	**Total**
1	70.0%	70.0%	
2	75.0%	85.0%	
3	80.0%	100.0%	
Company mean $\bar{X}_i$ and $\bar{X}$	75.0	85.0	80.0
Expected value of process variance *vi*	0.0025	0.0225	
Variance of hypothetical mean *a*			0.00417
$K_i = v_i/a$	0.00250/0.00417 =0.60	0.0225/0.00417 = 5.40	
$Z_i = n/(n+k)$	3/(3+0.60) = 83.33%	3/(3+5.4) = 35.7%	
$X_{Ei} = Z_i\bar{X} + (1 - Z_i)\mu$	75.7%	81.8%	

Note that company 1, which has much less variation in A/E ratios over the study period, has a lower expected value of process variance, and therefore higher credibility.

[17] *Loss Models: From Data to Decisions,* Example 5.36 and Introductory Credibility Theory, Example 4.3.2.

[18] *Loss Models: From Data to Decisions,* Section 5.5.1 and Introductory Credibility Theory Chapter 5.

The GACT approach attempts to obtain the variance components based on either model considerations (requiring similar information as in LFCT) or historical data (from which the variance components can be calculated without assumptions about models).

The difficulty with this approach lies in the data currently available for the industry. While companies can track their own A/E ratios by over time by subcategory, the problem of estimating the "between company" variation remains (no company has access to another's data).[19]

Many companies group mortality data for submission to experience studies, and grouped data is not sufficiently detailed to support the calculation of the parameter estimates. If seriatim data is not supplied to the experience study, modifications to the parameter estimates would be needed.

Buhlmann-Straub[20]

The Buhlmann model provides a simple, theoretically consistent formula but does not allow for variations in exposure or claim size. Buhlmann-Straub is a generalization of Buhlmann model that allows for variations in exposure or size.

Introduce the amount $m_{j,}$ a known constant that measures exposure i.e., m_j is expected claims (in $).

Assume $X_1, X_2, \ldots, X_n$ are independent conditional on Θ with common mean (as before). Then the hypothetical mean is given by

$$u(\theta) = \mathrm{E}\left(X_j \mid \Theta = \theta \right)$$

as before, but the conditional variances are

$$\mathrm{v}(\theta) = \mathrm{Var}\left(X_j \mid \Theta = \theta \right) = \mathrm{v}(\theta)/m_j$$

and

Z = m / (m+k)

where k is same as under the Buhlmann model above and m = sum of all exposure amounts m_j

This formula reflects variations in exposure and allows for between company effect and within company effect.

Development of the non-parametric estimators for the Buhlmann-Straub model is given in *Loss Models: From Data to Decisions*, subsection 5.5.1.

$$\hat{v}_i = \frac{\sum_{j=1}^{n} m_{ij}\left(X_{ij} - \bar{X}_i\right)^2}{n_i - 1}$$

$$\hat{v} = \frac{\sum_{i=1}^{r} \sum_{j=1}^{n_i} m_{ij}\left(X_{ij} - \bar{X}_i\right)^2}{\sum_{i=1}^{r} (n_i - 1)}$$

$$\hat{a} = \left(m - m^{-1}\sum_{i=1}^{r} m_i^2\right)^{-1}\left[\sum_{i=1}^{r} m_i(\bar{X}_i - \bar{X})^2 - \hat{v}(r-1)\right]$$

[19] One solution might be to have the CIA specify subcategories, and periodically publish the variation across companies by subcategory (using experience studies with specified subcategories).

[20] *Loss Models: From Data to Decisions,* subsection 5.4.4 and Introductory Credibility Theory, subsection 4.4.

Canadian Institute of Actuaries
360 Albert Street, Suite 1740
Ottawa, ON K1R 7X7
613-236-8196
head.office@cia-ica.ca
cia-ica.ca
seeingbeyondrisk.ca

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