Canadian Institute of Actuaries / Institut canadien des actuaires

Member's Paper

Selecting Discount Rates for Assessing Funded Status of Target Benefit Plans

By
Chun-Ming (George) Ma, PhD, FCIA, FSA

April 2018

Document 218041

Selecting Discount Rates for Assessing Funded Status of Target Benefit Plans

By
Chun-Ming (George) Ma, PhD, FSA, FCIA

Abstract	**Résumé**
For the purpose of determining the going concern funded status of a defined benefit pension plan, the current actuarial practice is to determine the liabilities of the plan using a discount rate based on the expected investment return from the pension fund (the "traditional" approach). On the other hand, financial economists have advocated the use of a discount rate based on the market yields of investment-grade bonds, with an appropriately low level of risk, whose cash flows reasonably match the expected benefit payments, regardless of how the plan assets are invested (the "financial economics" approach). This paper explores the issue of selecting discount rates for assessing the funded status of target benefit plans. A target benefit plan is a pension plan that aims to provide a target retirement income to its members through the pooling of economic and demographic risks, where the employer's funding obligation is predefined while members' benefits may be adjusted upwards or downwards relative to the target. From the viewpoint of managing the risk of intergenerational inequity, the paper proposes that the only discount rate for assessing the funded status of a target benefit plan that serves the best interests of members is one based on the traditional approach. To support this proposition, we conduct Monte Carlo simulations on three model	Lorsqu'il s'agit de déterminer le niveau de provisionnement d'un régime de retraite à prestations déterminées sur base de continuité, la méthode actuellement employée par les actuaires consiste à déterminer le passif du régime au moyen d'un taux d'actualisation basé sur le rendement prévu des placements de la caisse de retraite (la méthode « traditionnelle »). D'autre part, les économistes financiers préconisent l'utilisation d'un taux d'actualisation basé sur les rendements du marché des obligations de qualité supérieure, avec un niveau de risque suffisamment faible, et dont les flux monétaires correspondent assez bien aux paiements de prestations prévus, quelle que soit la manière dont les actifs du régime sont investis (la méthode de l'« économie financière »). Ce document porte sur le choix du taux d'actualisation pour évaluer le niveau de provisionnement des régimes à prestations cibles. Un régime à prestations cibles en est un qui cherche à procurer un revenu de retraite ciblé à ses participants par la mutualisation des risques économiques et démographiques et pour lequel l'obligation de provisionnement de l'employeur est prédéfinie, alors que les prestations des participants peuvent être rajustées à la hausse ou à la baisse par rapport à la cible. Du point de vue de la gestion du risque d'iniquité intergénérationnelle, il est proposé dans

plans to demonstrate the impact on pension wealth distributions resulting from the two discount rate approaches.	le document que le seul taux d’actualisation permettant d’évaluer le niveau de provisionnement d’un régime à prestations cibles qui puisse servir les intérêts des participants était celui basé sur la méthode traditionnelle. Pour soutenir cette proposition, nous effectuons des simulations de Monte Carlo portant sur trois régimes modèles afin de démontrer l’impact des deux méthodes sur la distribution du patrimoine retraite.

Table of Contents

1. Introduction

For the purpose of funding a defined benefit pension plan as a going concern, there are two broad approaches to setting the discount rate assumption for determining the liability for benefits promised to members (which may be referred to as a "funding target" in pension plan communications). The first is the *traditional* approach, which determines the liability by discounting the expected future benefit payments by the rate of investment return that the plan assets are expected to earn over the long term. The other is the *financial economics* approach, which determines a "market-consistent" value of the liability that is independent of the expected return of the plan assets. Financial economists apply this approach by noting that pension benefit payments have many similarities to bond cash flows, and that market value information of a bond's cash flow is readily available.

When applied to the funding of a defined benefit pension plan, the traditional approach to setting the discount rate assumption attempts to answer the following question: what amounts of contributions need to be made in advance to fund the promised benefits over the long term? On the other hand, the financial economics approach attempts to answer a different question: what is the market-consistent value of the plan's liability at a point in time, as determined by the market interest rates effective at that time? This involves finding a bond portfolio with characteristics (such as duration and risk) similar to the plan's benefit obligations, and then discounting the expected benefit cash flows using the yield curve underlying the bond portfolio. The discounted present value determined on this basis is considered to be a fair proxy for the cost of benefits based on observed market conditions.

In this paper, we explore the discount rate issue in the context of a target benefit plan (TBP). The CIA Task Force on Target Benefit Plans (Canadian Institute of Actuaries c. 2015) describes this plan design as follows:

> . . . a TBP is a collective, pre-funded pension plan pooling both economic and demographic risks, with a predefined retirement income goal (the "target benefit"), where the employer's financial liability is limited to predefined contributions while members' benefits may periodically be adjusted upwards or downwards relative to the original target.

To determine the funded status of a target benefit plan, which in turn determines when and how members' benefits will be adjusted, we argue that it is inappropriate to use a discount rate based on the financial economics approach. Doing so could lead to an unfair distribution of plan assets for the payment of benefits to members, either within a single generation or between different generations of members. It is suggested that, from an intergenerational equity viewpoint, the only basis for assessing the funded status of a target benefit plan that serves the best interests of plan members is one that reflects the expected return on the plan assets, namely, the traditional approach.

This paper is organized as follows. In section 2, we outline the current practice in Canada and the United States regarding the setting of discount rate assumptions for funding defined benefit plans. In section 3, we provide a general reasoning supporting our main proposition that a discount rate based on the traditional approach would provide a logical basis, from the perspective of managing the risk of intergenerational inequity, for assessing the funded status of a target benefit plan. In section 4, we describe the numerical analysis conducted on three simple model TBPs using Monte Carlo simulation technique. We observe in section 5 that use of the expected investment return as the discount rate would give rise to a fair distribution of plan assets for the payment of benefits to members, while the use of a discount rate based on the financial economics principles would not. In section 6, we show that intergenerational risk sharing can be applied to reducing benefit volatility, and in section 7 we conclude.

2. Current Practice in Setting Discount Rate Assumptions

According to current actuarial standards in Canada and the United States (Actuarial Standards Board 2013) (Canadian Institute of Actuaries a. 2015) (Canadian Institute of Actuaries b. 2015), a discount rate for determining the present value of future pension benefits may be based on the expected investment return from the pension fund (i.e., the traditional approach)[1]. Alternatively, the actuary may use a discount rate based on the market yields of investment-grade bonds whose cash flows reasonably match the benefits expected to be paid in the future, with an appropriately low level of risk, regardless of how the plan assets are invested (i.e., the financial economics approach). Except where a margin for adverse deviations is included, the discount rate assumption should incorporate no significant bias.

In Canada and the United States, it is a common practice to use a discount rate based on the traditional approach for determining the funding requirements or the funded position of private-sector and public-sector defined benefit pension plans. For example, this approach is widely used by US and local government pension plans. It is also generally supported by actuaries providing actuarial and consulting services to large multi-employer pension plans (Shapiro and Franklin 2013).

On the other hand, financial economists have advocated the use of market-consistent interest rates for the valuation of pension benefits. The riskiness of the benefits determines the discount rate for determining present values. For pure defined benefit pension plans that guarantee a lifetime pension to their beneficiaries, financial economists contend that, as the benefits are risk-free from the beneficiaries' viewpoints, the pension payments should be discounted against the term structure of “risk-free” interest rates to produce a market-consistent value of liabilities. A document titled Pension Actuary's Guide to Financial Economics (Joint AAA/SOA Task Force 2006), provides a comprehensive exposition in support of the financial economics principles. In examining the Dutch collective defined contribution pension system under which participants share risks by means of intergenerational smoothing, both Nijman et.al. (Nijman, et al. 2013) and Kocken (Kocken 2012) emphasize the importance of market-consistent valuation of benefit entitlements.

[1] If the expected fund return is established by means of a stochastic methodology, it would normally correspond to the median of the distribution of long-term investment returns of the pension fund based on the plan's investment policy.

3. The Main Proposition

Target benefit plans are collective pension arrangements combining certain design elements of traditional defined benefit (DB) and defined contribution (DC) pension plans, as described in the following table (Society of Actuaries b. 2016).

Elements of TBPs adopted from	
Traditional DC plans	**Traditional DB plans**
Predefined contribution levels	Predefined lifetime retirement benefits
Sponsor liability limited to contributions	Collective asset pool without individual accounts
Benefit level not guaranteed	Demographic risks (mortality in particular) pooled among plan members

In practice, TBPs span a wide range between more DC-like designs at one end of the spectrum, with frequent adjustments to benefits in response to emerging experience, and more DB-like designs at the other end, with an emphasis on the security and stability of target benefit delivery.

In this paper, we focus our analysis on a target benefit plan design that is more DC-like. Predefined contributions are paid into the plan over the career of plan members and invested in a pool of financial assets to provide a target retirement income. Periodic funding assessments are performed on the plan to determine if any benefit adjustment relative to the target benefit is required to ensure that the plan is financially sustainable. The assessment involves a comparison of the value of plan assets with the liability for the benefits targeted under the plan, resulting in either a funding shortfall or excess[2]. The discount rate to be used for the measurement of the liability is the main issue we intend to address.

Under the particular TBP design, financial and demographic risks to which the plan is exposed are shared among members (including pensioners, if applicable) by adjusting the benefit payable to each member by the same proportion, relative to the target benefit, such that there will be no funding shortfall or excess after adjustment[3]. This risk-sharing arrangement may be perceived to be inequitable if some members are expected to receive fewer benefits than others with identical attributes (e.g., age, service, salary, etc.) for the risk assumed (Gagne 2015) (Sanders 2014). Inequity may arise between individuals in the same membership class—e.g., pensioners who are living longer against those who die prematurely, young workers against those who are near retirement. Inequity may also appear between different classes of members—e.g., active members versus pensioners. For the plan to be sustainable, such risk of intergenerational or intra-generational inequity must be carefully managed. In particular, we

[2] If the value of plan assets is less (greater) than the liability for target benefits, a funding shortfall (excess) is said to exist.
[3] It is assumed that the benefit adjustment mechanism is well communicated to and understood by plan members.

need to find ways to avoid inequitable distributions of plan assets[4] for the payment of benefits to members.

In the case of a target benefit plan where any unexpected investment gains or losses are fully allocated to members by means of benefit adjustments (such as the one described above), we need to first establish an appropriate target benefit for the plan. We deal with this issue by considering the following basic equation of balance:

Contributions + investment earnings = benefit payments + plan expenses

This equation tells us immediately that, given the fixed contribution commitment under the plan and for a given level of expenses, the only way to offset the effect of any unanticipated investment losses (or gains) on the equation of balance is to lower (or increase) the benefit payments. The higher the investment earnings, the higher the amount of benefit payments the plan can provide. The target benefit level must be necessarily related to an expectation of future investment earnings.

How the benefit payments will be adjusted is a function of the returns that the plan assets will actually earn, not what we expect they will earn. But until those earnings are realized, the expected investment return is, arguably, our best guide for determining the target benefit level. This paper will show that use of a discount rate based on expected investment return for funding assessment is key to maintaining *ex ante* equity among plan members.

3.1 Discount rate for Post-retirement Periods (Decumulation Phase)

Consider a mix of equity and fixed-income assets held in a pool to support the payment of pensions to a group of annuitants with identical attributes (i.e., same sex, age, health condition, and equal paid premiums when entering the pool). The asset pool represents the annuitants' collective pension wealth. Longevity pooling enables annuitants to share idiosyncratic longevity risk among themselves (but not from systemic longevity risk). The mortality assumption used to value the annuitants' benefits would reflect the life expectancy of the annuitant group. Reserved assets not required to pay those annuitants who die after short life (relative to the group's life expectancy) would be used to provide the benefits for the long-lived annuitants. As discussed above, the target pension would be established based on the expected investment return on the asset pool.

Investment gains or losses, relative to the discount rate used to measure the annuitants' liability for funding assessment, could be shared among the annuitants in the group, in the form of improved or reduced pension payments (relative to the target pension). They could also be shared with annuitants who enter the pool at later times. If the actual rate of return on the asset pool is higher than the discount rate used for funding assessment, a gain will arise. The gain on the assets attributable to the current group of annuitants could be applied in one of the following two ways:

[4] The plan assets under a TBP consist of predefined contributions made over the members' career plus investment earnings thereon. They can be viewed as members' collective "pension wealth".

1. To improve the pension payments, on a go-forward basis (i.e., payments to be made on or after the valuation date), to the surviving annuitants in the current group only; or
2. To improve the pension payments, on a go-forward basis, to all surviving annuitants including annuitants who enter the pool in the future.

Under method 1, the gain would be amortized effectively over the remaining lifetime of the surviving annuitants in the current group, as an improvement to their pension payments. Annuitants who die earlier than others would leave their remaining share of the gain in the asset pool for the benefit of the surviving annuitants. Under method 2, the gain would be amortized over the remaining lifetime of the surviving annuitants in the current group, as well as those who enter the pool at future times. This means that annuitants not yet in the current group could potentially benefit from the gain attributable to the assets related to the current group.

Symmetrically, investment losses would be shared by surviving annuitants in the form of a reduction to their go-forward pension payments.

Under the above two methods of benefit adjustment, note that younger annuitants would bear a larger share of investment losses than older annuitants, as measured by the total expected amount of benefits being reduced. This may be justifiable on the following grounds:

- Younger annuitants would have more future opportunities than older annuitants, through favourable investment experience, to restore prior reductions that were applied to their go-forward pension payments; and
- Younger annuitants would be rewarded with a larger share of investment gains as a compensation for bearing the additional burden.

The choice of discount rates used to measure the liability for annuitants' pension benefits determines the rate at which their collective pension wealth would be spent down. When compared with the expected rate of investment return, use of a discount rate that strips out all or part of expected risk premiums that could potentially be earned by the asset pool would give rise to a lower initial pension than the target pension and a more gradual amortization of future investment gains or losses, other things being equal. As there would be a greater incidence of gains than losses when a discount rate significantly lower than the expected investment return is used, use of method 1 to amortize gains and losses would cause a greater proportion of the pension wealth related to the current group to be used to benefit the long-lived annuitants in the same group. If method 2 is used, future generations of annuitants would also benefit from the gains.

If the discount rate is set as the expected rate of return on the asset pool, gains and losses will have an equal likelihood to occur in the future *ex ante*. This would avoid the biased transfer of pension wealth that is noted with the use of a lower discount rate.

3.2 Discount Rate for Pre-retirement Periods (Accumulation Phase)

Consider a target benefit plan with the following characteristics:

- It requires the same rate of contributions be made for each member;
- It pays a lump-sum retirement benefit when a member retires; and
- All members join the plan at the same age.

The expected investment return on the plan assets would determine the target retirement benefit the plan provides. The higher the expected investment return, the higher would be the target retirement benefit, and vice versa.

To assess the funded status of a target benefit plan, the accrued liability for the benefits targeted under the plan should be established based on a level cost method that reflects the fixed contributions committed to under the plan (Ma 2016). Specifically, the accrued liability for the plan would be determined as the difference between the present value of future target benefits and the present value of future contributions based on the plan's predefined contribution rate. A gain revealed in a valuation could be applied to improve the retirement benefits payable to members (relative to the original target). Note that a gain would not only benefit the current members but would also benefit members who join the plan in the future. On the other hand, a loss could be shared among members by reducing the retirement benefits payable to them.

If the liability for a target benefit plan is determined using a discount rate that strips out all or part of risk premiums that could potentially be earned by the plan assets, gains due to investment would be more likely to occur than losses. For the target benefit plan described above, future generations of members would be more likely to receive a higher retirement benefit than their predecessors. This is a form of wealth transfer that is biased against older generations of members. On an *ex ante* basis, use of the expected investment return as the discount rate would avoid such biased transfer of pension wealth across different generations of members.

In summary, if any shock in financial market returns, both positive and negative, is fully absorbed by plan members through benefit adjustments relative to the plan's target benefit, use of a discount rate based on the expected investment return on plan assets (i.e., the traditional approach) would provide an unbiased liability measurement for assessing the funded status of the plan. This would avoid the effect of unfair wealth distributions among members caused by the use of a materially different discount rate. We will provide support to this proposition by performing Monte Carlo simulations on some model TBPs.

4. Monte Carlo Simulations

For simulation purposes, we assume annual valuations are performed at the beginning of each year. We use some simplified plan designs and assumptions on covered membership, and assume plan expenses are paid outside the plan. We also restrict ourselves to the investigation of one risk factor—the investment return—to demonstrate the impact on pension wealth distributions due to the use of different discount rates for liability measurement.

4.1 Model Plans

We construct three simple target benefit plans as follows.

TABLE 1: Model Target Benefit Plans

Model Plan	Key Features
Plan A	• Covers a closed group of pensioners: 100 individuals enter the plan at age 65 at inception; • All individuals are exposed to static mortality decrements; • At inception, the same premium is paid into the plan fund for each individual to provide for a life annuity due; and • Annuity amounts paid over the lifetime of annuitants are variable, depending on the discount rate assumption used for funding assessment and the investment performance of the plan fund, as described in section 4.4.
Plan B	• Covers an open group of pensioners: 100 individuals enter the plan at age 65 at inception and at each anniversary thereafter; • All individuals are exposed to static mortality decrements; • As each individual enters the plan, the same premium is paid into the plan fund to provide for a life annuity due; and • Annuity amounts paid over the lifetime of annuitants are variable, depending on the discount rate assumption used for funding assessment and the investment performance of the plan fund, as described in section 4.4.
Plan C	• Covers an open group of active members: 100 individuals join the plan at age 30 at inception and at each anniversary thereafter; • All members retire at age 65 and none of them terminates or dies before retirement; • The same amount is contributed for each member at the beginning of each year from age 30 to age 64 inclusive; and • Benefits are paid as a lump sum to members at retirement and are variable depending on the discount rate assumption used for funding assessment and the investment performance of the plan fund, as described in section 4.4.

4.2 Asset Model

For each of the plans described in table 1, it is assumed that the plan assets are invested in Canadian stocks and Canada long bonds with a 50-50 mix. The investment portfolio is rebalanced on a regular basis to maintain a static asset allocation of 50-50 mix.

We derive the mean return A and variance V of the pension fund portfolio from the following Canadian economic statistics:

Table 2: Average Return, Standard Deviation, and Correlation

Asset class	Average return (%)	Standard deviation (%)	Correlation
Canadian stocks	5.13	18.21	-0.55
Canada long bonds	6.96	7.41	

Source: *CIA Report on Canadian Economic Statistics, table 2A* (Canadian Institute of Actuaries d. 2016) [5]

The results[6] are

$$A = 0.06045; \;\; V = 0.00595$$

It is assumed that future years' distributions of fund returns are independent and identically distributed (IID)[7]. Define the return factor $1 + R$ over any one-year period as e^r, where the random rate of return r is assumed to follow a normal distribution with mean μ and standard deviation σ (i.e., the return factor $1 + R$ is assumed to follow a log-normal distribution). We may write $1 + R = \exp(\mu + \sigma Z)$, where the random variable Z follows a standard normal distribution, i.e., $Z \sim N(0,1)$. The parameters μ and σ of the normal distribution are calculated from A and V as follows, see (Mindlin 2013):

$$\sigma = [\ln(1 + V(1 + A)^{-2})]^{1/2} = 0.0726 \tag{1}$$

$$\mu = \ln(1 + A) - \frac{1}{2}\sigma^2 = 0.056 \tag{2}$$

4.3 Liability Model

This section describes the liability model used for assessing the funded status of the three model plans.

---

[5] Average returns and standard deviations are taken over the period 2001–2015 and the correlation is taken over the period 2006–2015. The objective of our simulations is not to forecast future economic outcomes, but to demonstrate the wealth distribution effects resulting from the use of different discount rates for liability measurement. Therefore, we made no attempts to calibrate the asset model to the current economic environment.

[6] Let:

m_i be the average return of asset class i
s_i be the standard deviation of return of asset class i
c_{ij} be the correlation coefficient between asset classes i and j

For a portfolio of asset classes with weights $\{w_i\}$, such that $\sum_{i=1}^{n} w_i = 1$, the portfolio mean return A and variance V are calculated as follows:

$$A = \sum_{i=1}^{n} w_i m_i$$

$$V = \sum_{i,j=1}^{n} w_i w_j s_i s_j c_{ij}$$

[7] The IID assumption is overly simplified in that it does not incorporate the dynamics of observed "real-world" return patterns. However, it permits the representation of portfolio return as a normally distributed random variable. This facilitates the comparison of pension wealth distributions resulting from different discount rate assumptions. We consider it to be an acceptable assumption for the purpose of this paper.

The pensioners' target pension is normalized to 1. It is assumed that all pensioners are exposed to static mortality decrements according to the following life table:

Table 3: Life Table

Age (x)	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
# of survivors (l_x)	100	98.8	97.4	95.9	94.3	92.6	90.8	88.8	86.7	84.5	82.1	79.5	76.8	73.8	70.6

Age (x)	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
# of survivors (l_x)	67.1	63.3	59.3	55.1	50.8	46.5	42.2	37.9	33.6	29.4	25.4	21.5	17.9	14.6	11.6

Age (x)	95	96	97	98	99	100
# of survivors (l_x)	9.0	6.8	5.0	3.6	2.5	0.0

We use the following notation to develop the equations throughout section 4:

P^T	Target pension for pensioners
B^T	Target retirement benefit for active members
r_t	Simulated fund rate of return in period $[t, t+1)$
R_t	Simulated return in period $[t, t+1)$, $R_t = e^{r_t} - 1$
P_t	Simulated pension at time t
B_t	Simulated retirement benefit at time t
$\bar{\bar{L}}_t$	Expected liability at time t based on target pension P^T or target retirement benefit B^T
L_t	Simulated liability at time t based on simulated pension or simulated retirement benefit
$\bar{P}^{(k)}$	Simulated average pension received over the lifetime of the k^{th} pensioner cohort
$B^{(k)}$	Simulated retirement benefit received by the k^{th} active cohort at retirement
G^T	Discount rate upon which target pension or target retirement benefit is based. It is set as the expected fund rate of return. For the purpose of simulations, it is taken as the average return of the asset model described in section 4.2
G	Discount rate used to calculate present value of expected future benefits, assumed to be fixed throughout simulation periods
$\ddot{a}_x$	Life annuity due of 1 at age x, based on discount rate G except otherwise noted
$\ddot{a}_{\overline{n}\vert}$	n-year term certain annuity due of 1, based on discount rate G except otherwise noted
C	Premium paid into either plan A or plan B to provide a life annuity due at age 65 for each pensioner

Plan A

The target pension P^T is normalized to 1, and the amount C is calculated as $\ddot{a}_{65}$ based on discount rate G^T.

The expected liability at time t based on target pension P^T is calculated as follows:

$$\bar{L}_t = P^T l_{65+t}\ddot{a}_{65+t}\,,\ t \leq 34 \tag{3}$$

Initial pension is given by

$$P_0 = C/\ddot{a}_{65} \tag{4}$$

where $\ddot{a}_{65}$ is calculated based on discount rate G.

The simulated liability at time t based on simulated pension P_t is calculated as

$$L_t = P_t l_{65+t}\ddot{a}_{65+t}\,,\ t \leq 34 \tag{5}$$

The average pension received over the lifetime of a pensioner who dies after time t but before time $t+1$ is equal to

$$\bar{P}_t = \frac{(\sum_{k=0}^{t} P_k)}{t+1}, \qquad t \leq 34 \tag{6}$$

Plan B

Same as for plan A, the target pension P^T is normalized to 1 and the amount C is calculated as $\ddot{a}_{65}$ based on discount rate G^T.

The expected liability at time t based on target pension P^T is calculated as follows:

$$\bar{L}_t = \begin{cases} P^T\left(\sum_{k=0}^{t} l_{65+k}\cdot\ddot{a}_{65+k}\right), & t \leq 34 \\ P^T\left(\sum_{k=0}^{34} l_{65+k}\cdot\ddot{a}_{65+k}\right), & t > 34 \end{cases} \tag{7}$$

Initial pension is given by equation (4) based on discount rate G, same as for plan A. The simulated liability at time t based on simulated pension P_t is calculated as follows:

$$L_t = \begin{cases} P_t\left(\sum_{k=0}^{t} l_{65+k}\cdot\ddot{a}_{65+k}\right), & t \leq 34 \\ P_t\left(\sum_{k=0}^{34} l_{65+k}\cdot\ddot{a}_{65+k}\right), & t > 34 \end{cases} \tag{8}$$

The average pension received over the lifetime of the $(k+1)^{th}$ pensioner cohort, $k \geq 0$, is given by

$$\bar{P}^{(k+1)} = \frac{\sum_{h=0}^{34} P_{k+h} l_{65+h}}{\sum_{h=0}^{34} l_{65+h}} \tag{9}$$

Plan C

The annual contribution for each member is normalized to 1. The target retirement benefit payable to a member who retires at age 65 is given by

$$B^T = \ddot{a}_{\overline{35|}}(1+G^T)^{35} \tag{10}$$

where $\ddot{a}_{\overline{35|}}$ is based on discount rate G^T. At $G^T = e^{\mu} - 1 = e^{0.056} - 1 = 5.76\%$, B^T is calculated to be 112.

The expected liability at time t based on target retirement benefit B^T is calculated using a level cost method as follows:

$$\bar{L}_t = \begin{cases} 100\left[\sum_{k=0}^{t}\left(\frac{B^T}{(1+G)^{35-k}} - \ddot{a}_{\overline{35-k|}}\right)\right], & t \leq 34 \\ 100\left[\sum_{k=0}^{34}\left(\frac{B^T}{(1+G)^{35-k}} - \ddot{a}_{\overline{35-k|}}\right)\right], & t > 34 \end{cases} \tag{11}$$

Likewise, the simulated liability at time t based on simulated retirement benefit B_t is calculated as

$$L_t = \begin{cases} 100\left[\sum_{k=0}^{t}\left(\frac{B_t}{(1+G)^{35-k}} - \ddot{a}_{\overline{35-k|}}\right)\right], t \leq 34 \\ 100\left[\sum_{k=0}^{34}\left(\frac{B_t}{(1+G)^{35-k}} - \ddot{a}_{\overline{35-k|}}\right)\right], t > 34 \end{cases} \tag{12}$$

The retirement benefit received by the $(k+1)^{th}$ cohort at retirement, $B^{(k+1)}$, is equal to B_{k+34}, for $k \geq 0$.

4.4 Benefit Adjustment Mechanism

As the pension fund is exposed to investment return risk, the fund balance at any future date may fall short of or exceed the amount required to support the target benefit. It is assumed that any unexpected investment gains or losses will be allocated to plan participants by applying an adjustment to their benefits, such that there will be no funding shortfall or excess after adjustment.

Plan A

At inception, 100 pensioners enter the plan and each contributes an amount C to pay for a life annuity due of 1. The amount C is calculated to be 11.314, based on a discount rate equal to the expected investment return from the pension fund G^T (i.e., 5.76%). Thus, the initial balance of the pension fund is equal to $F_0 = 100C = 1{,}131$.

For $t > 0$, the recursive formula of pension fund balance F_t is given by

$$F_t = (F_{t-1} - l_{65+t-1} \cdot P_{t-1})(1 + R_{t-1}) \tag{13}$$

At time t, the pension P_t is derived from the target pension P^T using the following adjustment formula:

$$P_t = P^T \left(\frac{F_t}{\overline{L}_t}\right) \tag{14}$$

This adjustment formula effectively amortizes any unexpected investment gain or loss, relative to the discount rate assumption, over the remaining lifetime of surviving pensioners. The lower the discount rate assumption, the slower will be the amortization.

Plan B

The initial balance of the pension fund for this plan is the same as that for plan A, i.e., $F_0 = 1{,}131$.

For $t > 0$, the recursive formula of pension fund balance F_t is given by

$$F_t = \begin{cases} \left(F_{t-1} - \sum_{k=0}^{t-1} l_{65+k} \cdot P_{t-1}\right)(1 + R_{t-1}) + l_{65} \cdot C, & t \leq 35 \\ \left(F_{t-1} - \sum_{k=0}^{34} l_{65+k} \cdot P_{t-1}\right)(1 + R_{t-1}) + l_{65} \cdot C, & t > 35 \end{cases} \tag{15}$$

At time $t > 0$, the pension P_t is derived from the target pension P^T using the following formula:

$$P_t = P^T \left(\frac{F_t}{\overline{L}_t}\right) \tag{16}$$

This adjustment formula effectively amortizes any unexpected investment gain or loss, relative to the discount rate assumption, over the remaining lifetime of surviving pensioners (including new pensioners who just enter the plan at time t). The lower the discount rate assumption, the more gradual will be the amortization.

Plan C

The members' annual contribution rate is normalized to 1. The initial balance of the pension fund is 0 at plan inception.

For $t > 0$, the recursive formula of pension fund balance F_t is given by

$$F_t = \begin{cases} (F_{t-1} + 100)(1 + R_{t-1}), & t \leq 34 \\ (F_{t-1} + 100)(1 + R_{t-1}) - 100B_{t-1}, & t > 34 \end{cases} \quad (17)$$

At time $t > 0$, the retirement benefit B_t is derived from the target retirement benefit B^T using the following formula:

$$B_t = \begin{cases} B^T \left(\dfrac{F_t + 100 \sum_{k=0}^{t} \ddot{a}_{\overline{35-k}|}}{\bar{L}_t + 100 \sum_{k=0}^{t} \ddot{a}_{\overline{35-k}|}} \right), & t \leq 34 \\ B^T \left(\dfrac{F_t + 100 \sum_{k=0}^{34} \ddot{a}_{\overline{35-k}|}}{\bar{L}_t + 100 \sum_{k=0}^{34} \ddot{a}_{\overline{35-k}|}} \right), & t > 34 \end{cases} \quad (18)$$

Under the above adjustment formula, any investment gain or loss, relative to the discount rate assumption, is shared among all active members (including new members who just enter the plan at time t). For a given amount of investment gain or loss, the lower the discount rate assumption, the more gradual will be the adjustment to the retirement benefit.

4.5 Evaluation Measures

To demonstrate how the choice of discount rate may impact the benefits paid to different generations of members in a target benefit plan, we focus our analysis on the following two discount rates[8]:

- A discount rate based on the traditional approach, being equal to the average return of the asset model described in section 4.2, i.e., G is set as $G^T = e^{\mu} - 1 = 5.76\%$; and
- A discount rate that strips out any expected risk premiums that could potentially be earned by the pension fund, being equal to 2.5%[9]. For convenience, we shall refer to this rate as the "risk-free" rate.

For plan A simulation, the simulation period is the maximum life span of the covered pensioners, i.e., 35 years. For plan B and plan C simulations, the chosen number of years of simulation is 100.

A target pension is set as 1 for both plan A and plan B simulations. The initial pension P_0 based on 5.76 percent discount rate is the same as the target pension of 1. When a discount rate of 2.5 percent is used, the funded ratio of the plan (i.e., the ratio of the value of pension fund assets to the liability for target pensions) drops down to 0.76 at the outset. Thus, the initial pension for pensioners must be immediately reduced to 0.76 to eliminate the funding shortfall.

---

[8] In a recent research report released by the Society of Actuaries (Society of Actuaries a. 2016), the researchers applied stochastic modelling to investigate how different design features and funding strategies would impact the performance of target benefit plans over the short- and long-term. The researchers observed how the choice of discount rate would affect the benefit outcomes of different cohorts of retirees, but stopped short of providing insights into the appropriateness of the discount rates applied in the modelling from the intergenerational fairness viewpoint.

[9] We did not use a stochastic model for interest rates to set this discount rate. Instead, we assume that the average (geometric) return on safe fixed-income assets (e.g., government bonds) over the long term is 2.5 percent.

For plan C simulation, the target retirement benefit is set as 112. The initial retirement benefit B_0 based on 5.76 percent discount rate is the same as the target retirement benefit of 112, and that based on 2.5 percent discount rate must be immediately reduced to 56 to eliminate the funding shortfall at plan inception.

The measures used to analyze the wealth distribution effects resulting from the use of different discount rates are set out in the following table.

Table 4: Measures for Analyzing Wealth Distribution Effects

Simulation for:	Evaluation measures
Plan A	1. Distribution of average pension paid over the lifetime of each pensioner, by age at death. 2. Probability of average pension falling below 1.0, 0.9, and 0.8, by age at death. 3. Mean, median, standard deviation, skewness[10], and kurtosis[11] for the distribution of average pension, for selected ages at death.
Plan B	1. Distribution of average pension paid over the lifetime of pensioners, by cohort. 2. Probability of average pension falling below 1.0, 0.9, and 0.8, by cohort. 3. Mean, median, standard deviation, skewness, and kurtosis for the distribution of average pension, for selected cohorts. 4. Distribution of pension fund balance, by year of simulation.
Plan C	1. Distribution of retirement benefit paid to members, by cohort. 2. Probability of retirement benefit falling below 100 percent, 90 percent, and 80 percent of target retirement benefit, by cohort. 3. Mean, median, standard deviation, skewness, and kurtosis for the distribution of retirement benefit, for selected cohorts.

Note from the above table that we use measures based on average pension for plan A and plan B simulations. To demonstrate the effect of pension wealth distributions, we consider that it is more meaningful to compare the average pension paid over the lifetime of pensioners, either by age at death or by cohort, as opposed to pensions paid at specific times.

5. Pension Wealth Distributions among Members

From the simulation outputs, we have developed the various evaluation measures described in section 4.5. A summary of the results is provided in appendix A. Key observations and commentaries on pension wealth distributions due to the use of two different discount rates for liability measurement, i.e., expected rate of return and risk-free rate, are as follows.

[10] Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the centre point. The skewness for a normal distribution is zero. Negative values for the skewness indicate data that are skewed left and positive values for the skewness indicate data that are skewed right.

[11] Kurtosis is a measure of whether the data are heavy-tailed or light-tailed relative to a normal distribution. The kurtosis for a standard normal distribution is three. A kurtosis of greater than three indicates a heavy-tailed distribution and a kurtosis of less than three indicates a light-tailed distribution.

5.1 Plan A Simulation

From the summary results presented in appendix A.1 (see figures 4 and 5 and table 6), we can make the following observations.

- When using the expected rate of return as the discount rate for liability measurement:
  - The pension starts at the target level of 1. The median of average pension paid to pensioners also stays close to 1, regardless of their age at death.
  - The range of average pension to pensioners who die at old ages is wider than those who die at young ages.
  - The mean of average pension is somewhat higher than the median value, as shown in table 6, for selected ages at death. This asymmetry is due to the effect on pension adjustment associated with investment gains (relative to the discount rate) being greater than that associated with investment losses.
  - As indicated by the skewness measure, the distribution of average pension is skewed more right as the age at death increases.
  - The probability of average pension falling below 1.0 declines gradually from around 0.5 at extreme young ages at death to around 0.43 at extreme old ages at death. On the other hand, the probability of average pension falling below 0.8 or 0.9 increases from close to 0 at extreme young ages at death to 0.16 and 0.28, respectively, at extreme old ages at death.
- Using the risk-free rate as the discount rate for liability measurement changes the distribution of pension wealth among pensioners considerably:
  - The pension starts at 0.76, well below the target pension of 1. The median of average pension increases as the age at death increases, reaching 1.37 to those pensioners who die at the terminal age of 100. The median stays below 1 before age 82 and is above 1 thereafter.
  - The range of average pension expands and shifts upwards as the age at death increases.
  - Both of the mean and median of average pension increase appreciably as the age at death increases, as shown in table 6 for selected ages at death.
  - Other statistics such as standard deviation, skewness, and kurtosis shown in table 6 do not appear to be materially different from those associated with the use of expected return discount rate.
  - The probability of average pension falling below 1.0, 0.9, or 0.8 declines from almost a certainty at extreme young ages at death to a very low level (less than 0.2) at extreme old ages at death.

Commentary: The choice of discount rate for funding assessment clearly affects the distribution of pension wealth among pensioners who participate in the sharing of investment risk. When a discount rate equal to the expected rate of return from the pension fund is used, the distribution is *ex ante* fair in that all pensioners paying the same amount into the pension fund can expect to receive the same target pension, regardless of their age at death. On the other hand, when a significantly lower discount rate such as the risk-free rate is used, the distribution would benefit pensioners who live a long life at the expense of pensioners who die prematurely. Long-lived pensioners can expect to receive a level of pension that is appreciably higher than predeceased pensioners.

5.2 Plan B simulation

From the summary results presented in appendix A.2 (see figures 7, 8, and 9 and table 7), we can make the following observations.

- When using the expected rate of return as the discount rate for liability measurement:
    - The median of average pension to different cohorts of pensioners is all close to 1.
    - The range of average pension to later cohorts is wider but there is very little change after the 25th cohort or so.
    - The probability of average pension falling below 1.0 does not vary much between cohorts. Its value falls in a narrow range between 0.42 and 0.45. On the other hand, the probability of average pension falling below 0.8 or 0.9 increases from close to 0 for the first cohort to around 0.05 and 0.18, respectively, for the 25th cohort. These probabilities remain at about the same level for subsequent cohorts.
    - The pension fund balance grows yearly from plan inception as new pensioners join the plan. The median of pension fund balance increases to about 15,800 after 35 years of plan operation, and remains at that level subsequently. The range of pension fund balance, from the fifth percentile to the 95th percentile, is virtually unchanged after 35 years.
- Using the risk-free rate as the discount rate for liability measurement changes the distribution of pension wealth among pensioners considerably:
    - The median of average pension paid to the first cohort is about 0.89, which is lower than the target pension of 1. The median of average pension to subsequent cohorts increases steadily, reaching the level of 1.20 to the 50th cohort.
    - The mean of average pension to successive cohorts also increases steadily, as shown in table 7.
    - The range of average pension to successive cohorts expands and shifts upwards over time.
    - The standard deviation for the distribution of average pension to later cohorts is noticeably higher than that associated with the use of expected return discount rate.
    - The probability of average pension falling below 1.0 declines from 0.9 for the first cohort to 0.14 for the 50th cohort. There is a very low probability of average pension falling below 0.9 or 0.8 for later cohorts, being less than 0.07 and 0.03 respectively.

    - The pension fund balance grows from plan inception at a faster pace than that associated with the expected return discount rate, as lower amounts of pensions are paid to earlier cohorts of pensioners. The median of pension fund balance increases to about 22,000 after 35 years of plan operation, and continues to grow, asymptotically, towards 23,000, a level much higher than the 15,800 in the case of expected return discount rate. A higher amount of assets is accumulated in this instance to support the higher pensions paid to later cohorts of pensioners.

Commentary: The choice of discount rate for funding assessment clearly affects the distribution of pension wealth between different generations of pensioners. When a discount rate equal to the expected rate of return from the pension fund is used, the distribution is *ex ante* fair in that pensioners paying the same amount into the pension fund can expect to receive the same target pension, regardless of when they enter into the plan. On the other hand, if a significantly lower discount rate such as the risk-free rate is used, the distribution would benefit later cohorts of pensioners at the expense of earlier cohorts. Later cohorts can expect to receive a higher level of pension, which is financed in part by transfers of pension wealth from earlier cohorts. This kind of wealth transfer should be avoided if the plan's objective is to maintain equity among different generations of members.

5.3 Plan C Simulation

From the summary results presented in appendix A.3 (see figures 11 and 12 and table 8), we can make the following observations.

- When using the expected rate of return as the discount rate for liability measurement:
    - The median of retirement benefit to different cohorts of members at retirement is close to the target level of 112.
    - The range of retirement benefit for successive cohorts expands gradually over time.
    - The probability of retirement benefit falling below 100 percent of target does not vary much between cohorts. Its value falls in a narrow range between 0.40 and 0.44. On the other hand, the probability of retirement benefit falling below 80 percent or 90 percent of target increases from 0.13 and 0.26 for the first cohort to around 0.23 and 0.32, respectively, for the 35th cohort. These probabilities remain at about the same level for subsequent cohorts.
    - The mean of retirement benefit for the later cohorts is noticeably higher than the median, as shown in table 8. This asymmetry is due to the effect on benefit adjustment associated with investment gains (relative to the discount rate) being greater than that associated with investment losses.
- Using the risk-free rate as the discount rate for liability measurement changes the retirement benefits paid to different cohorts of members considerably:
    - The median of retirement benefit paid to the first cohort is about 79, which amounts to only 70 percent of the target (112). The median of retirement

benefit to subsequent cohorts increases steadily, reaching 187 for the 50th cohort.

- The mean of retirement benefit to successive cohorts also increases steadily, as shown in table 8.
- The range of retirement benefit to successive cohorts expands and shifts upwards over time.
- The standard deviation for the distribution of retirement benefit to later cohorts is noticeably higher than that associated with the use of expected return discount rate.
- The distribution for later cohorts is skewed more right than earlier cohorts and exhibits a clear upside potential. For example, there is a 25 percent probability that the retirement benefit to the 50th cohort could exceed 258, which is 2.3 times the target.
- The probability of retirement benefit falling below 100 percent of target declines from almost a certainty for the first cohort to 0.17 for the 50th cohort. Similarly, the probability of retirement benefit falling below 80 percent or 90 percent of target also declines precipitously from the earlier cohorts to the 35th cohort or so, and settles eventually at the level of 0.12 and 0.15, respectively.

Commentary: The choice of discount rate for funding assessment clearly affects the distribution of pension wealth for the payment of retirement benefits to different generations of members. When a discount rate equal to the expected rate of return from the pension fund is used, the distribution is *ex ante* fair in that members with the same career contributions can expect to receive the same retirement benefit, regardless of when they enter into the plan. On the other hand, if a significantly lower discount rate such as the risk-free rate is used, the distribution would benefit later cohorts of members at the expense of earlier cohorts. Later cohorts can expect to receive a significantly higher level of retirement benefit, which is financed in part by transfers of pension wealth from earlier cohorts.

6. Volatility of Pension Payments

Members' benefits under a DC-like target benefit plan are adjusted upwards or downwards relative to the predefined target, in direct response to the emerging experience of the plan. Members may be unwilling to participate in the plan if benefit payments fluctuate significantly from year to year. Benefit volatility can be controlled through the adoption of certain plan design features (e.g., “no-action” range for benefit adjustment) and prudent funding policies that develop and maintain adequate funding cushions. For open plans with an ongoing entry of new members, we demonstrate that intergenerational risk sharing can be an effective means to manage this risk.

Recall that plan A covers a closed group of pensioners who enter the plan at age 65 paying the same premium into the plan for the benefit of a life annuity due. Any unexpected investment gains or losses are allocated to the surviving pensioners through an adjustment to pension payments according to equation (14). In essence, each year's gains or losses are amortized over the remaining lifetime of surviving pensioners, the period of which decreased from 19.2 years at plan inception to just one year at year 35, as shown in figure 1 below.

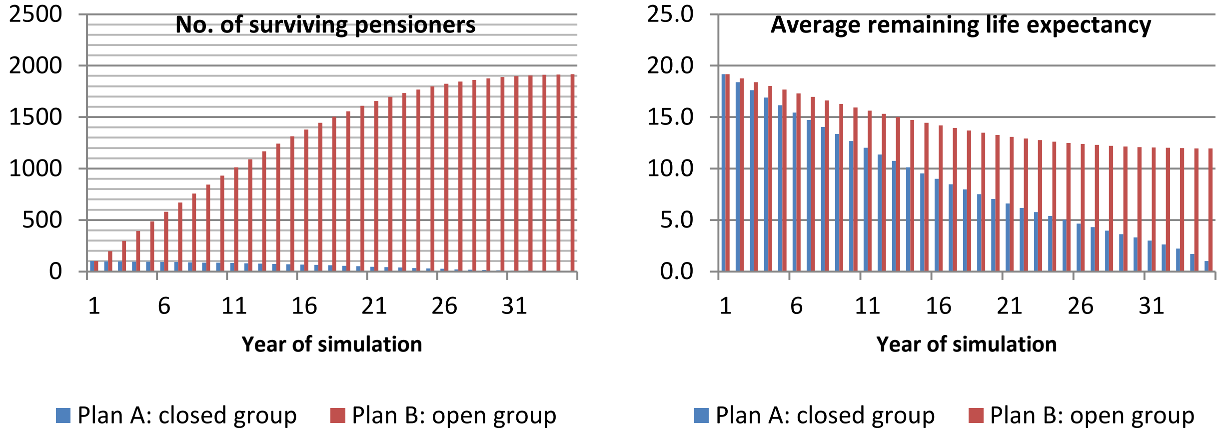


Figure 1: Amortization base – number of surviving pensioners and average remaining life expectancy

On the other hand, plan B is open to future pensioners who also join the plan at age 65 and pay the same premium into the plan for a life annuity due. Any unexpected investment gains or losses are applied as an adjustment to pension payments according to equation (16). In this case, any gains or losses are effectively amortized over the remaining lifetime of surviving pensioners, including those who join the plan after plan inception. As shown in figure 1, with the joining of new pensioners each year,

- There are progressively more pensioners participating in risk sharing (increased from 100 at plan inception to 1,916 after 35 years); and
- The average remaining life expectancy for surviving pensioners would decrease more gradually (from 19.2 years at plan inception to 12 years after 35 years).

This broadened amortization base has the effect of reducing the extent of pension adjustments that may otherwise be required in a closed-group situation.

Figure 2 compares the distribution (5th, 25th, 50th, 75th, and 95th percentiles) of yearly pension payments, by year of simulation, between plan A and plan B, based on a discount rate equal to the expected rate of fund return. Relative to plan A, the variability of yearly pension payments in plan B is greatly reduced as a result of intergenerational diversification.

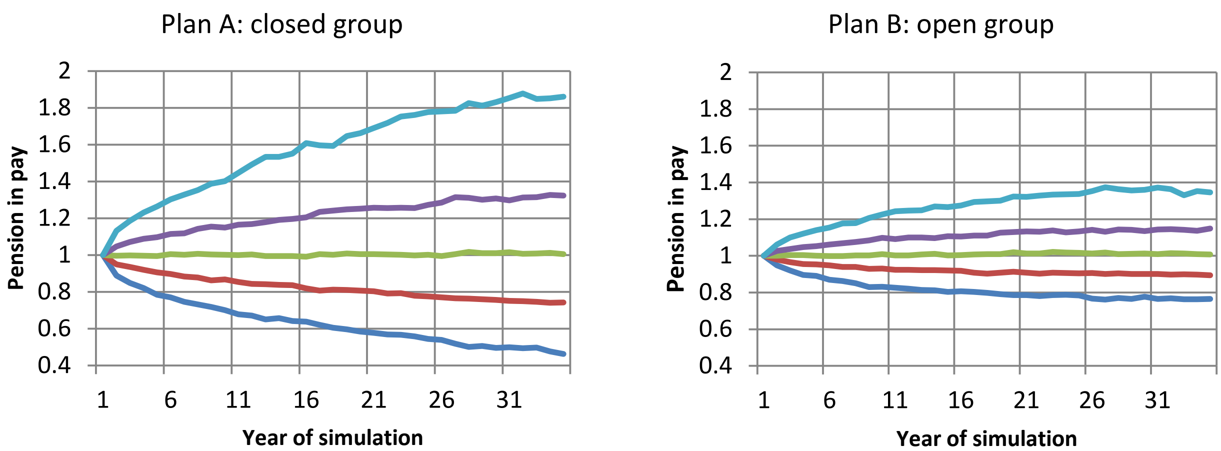


Figure 2: Distribution of yearly pension payment, by year of simulation

Table 5 below compares key statistics for the distribution of yearly pension payments for selected years of simulation, between plan A and plan B. As can be seen, the standard deviations for plan B are significantly lower than plan A. This demonstrates that, in the case of open plans, intergenerational risk sharing can be an effective tool for managing the risk of benefit volatility.

Table 5: Key statistics for distribution of yearly pension payment

Plan A: closed group

Year of simulation	**6**	**11**	**16**	**26**
Mean	1.01	1.02	1.04	1.06
Median	1.01	1.00	0.99	1.00
Standard deviation	0.16	0.23	0.30	0.40
Skewness	0.44	0.57	0.87	1.06
Kurtosis	0.59	0.29	1.20	2.30

Plan B: open group

Year of simulation	**6**	**11**	**16**	**26**
Mean	1.01	1.01	1.02	1.03
Median	1.00	1.00	1.00	1.01
Standard deviation	0.09	0.13	0.14	0.18
Skewness	0.28	0.47	0.54	0.60
Kurtosis	-0.02	0.50	0.63	0.67

7. Conclusion

We have explored the issue of selecting discount rate assumptions for assessing the funded status of target benefit plans, from the perspective of managing intergenerational equity risk. We apply the following two approaches to DC-like TBPs, where any unexpected investment gains and losses are fully allocated to plan members through adjustments to their target benefits:

- The *traditional* approach that determines the liability for target benefits by discounting expected future benefits by the expected rate of investment return from the plan assets; and

- The *financial economics* approach that sets a discount rate by excluding expected risk premiums that could potentially be earned by the plan assets.

If the objective of a TBP is to distribute members' collective pension wealth for the payment of benefits to members on an equitable basis, we propose by general reasoning that it is inappropriate to use a discount rate developed from the financial economics approach for determining the funded status of the plan. Doing so could lead to an unfair distribution of pension wealth among members, either intra-generational or intergenerational. Use of a discount rate developed from the traditional approach would overcome this shortcoming.

We have conducted Monte Carlo simulations on three simple model plans to provide support to our proposition. When a risk-free discount rate is used, it can be seen from the simulation results for plan B and plan C (see appendices A.2 and A.3) that

- Earlier generations of members would have a lower expected benefit than later generations; and
- Both the level and security of benefits for later generations would be significantly improved from their predecessors.

The higher benefits expected to be received by later generations are financed in part by the pension wealth transferred from earlier generations.

Finally, for target benefit plans that are open to the joining of new members, we show that intergenerational risk sharing can be an effective means for managing the risk of benefit volatility.

Areas for Further Research

This paper considers the selection of discount rate assumption from the viewpoint of managing intergenerational equity risk. It is recognized that TBP designs span a wide spectrum with varying objectives on the level and security of benefits, stability of benefit payments, etc. (Society of Actuaries a. 2016) (Society of Actuaries b. 2016) (Society of Actuaries c. 2016) . The plan's design objectives would impact the choice of discount rate assumption for funding assessment.

This author believes that use of Monte Carlo simulation technique is a direct approach to studying the stochastic properties of benefit payments under TBPs, which can help to shed light on the issues faced by TBP stakeholders. For instance, if a plan aims to deliver a target benefit with a high level of certainty (e.g., 95 percent confidence)[12], plan stakeholders will need to address a number of design and management issues including the following:

- Given the plan's predefined contributions, what level of target benefit would be appropriate and acceptable to plan members?
- What investment strategy should be adopted given the plan's target benefit level and risk tolerance limits?

[12] See for example, the New Brunswick shared risk plan model (New Brunswick 2012).

- What margin (relative to the expected investment return) should be incorporated in the setting of discount rate assumption in order to achieve the desired level of security?
- Is the wealth distribution effect inherent in the plan design understood by and acceptable to plan members?

TBPs may adopt different triggers, smoothing mechanisms, and priorities for benefit adjustment. How would they affect the benefit payment pattern (in terms of level and volatility) and the pension wealth distribution between generations? How would they affect the probability of success in delivering the target benefit?

These are examples of issues that warrant further research.

Bibliography

Actuarial Standards Board. “Actuarial Standard of Practice No. 27 – Selection of Economic Assumptions for Measuring Pension Obligations.” September 2013. http://www.actuarialstandardsboard.org/wp-content/uploads/2014/02/asop027_172.pdf

Canadian Institute of Actuaries a. “Standards of Practice – Pensions.” June 2015. http://www.cia-ica.ca/docs/default-source/standards/sp060915e.pdf

Canadian Institute of Actuaries b. “Revised Educational Note – Determination of Best Estimate Discount Rates for Going Concern Funding Valuations.” December 2015. http://www.cia-ica.ca/docs/default-source/2015/215106e.pdf

Canadian Institute of Actuaries c. “Report of the Task Force on Target Benefit Plans.” 2015.

Canadian Institute of Actuaries d. “Report on Canadian Economic Statistics 1924–2015.” 2016.

Gagne, Claudia. “Closing the Generations Gap: Managing Intergenerational Equity Risk.” *CIA - Seeing Beyond Risk*, 2015.

Joint AAA/SOA Task Force . “Pension Actuary's Guide to Financial Economics.” *Society of Acctuaries.* 2006.
https://www.soa.org/professional-interests/pension/research-thinking-ahead/pen-finance-resources.aspx

Kocken, Theo. “Pension Liability Measurement and Intergenerational Fairness: Two Case Studies.” *Rotman International Journal of Pension Management*, 2012.

Ma, Chun-Ming. “An Actuarial Balance Sheet Approach to Assessing Sustainability of Target Benefit Plans.” *IPEBLA, IACA, PBSS, & IAAHS Joint Conference.* St. John's, Newfoundland, 2016.

Mindlin, Dimitry. “Present Values, Investment Returns and Discount Rates.” *CDI Advisors LLC*, May 2013.

New Brunswick. *Shared Risk Plans Regulation, NB Reg 2012-75.* 2012. https://beta.canlii.org/en/nb/laws/regu/nb-reg-2012-75/latest/nb-reg-2012-75.html

Nijman, Theo E., Stephan M. van Stalborch, Johannes A.C. van Toor, and Bas J.M. Werker. “Formalizing the New Dutch Pension Contract.” *Netspar*, January 2, 2013.

Sanders, Barbara. “Intergenerational Equity in Target Benefit Plans.” *Plans & Trusts*, July/August 2014: 18-22.

Shapiro, Josh, and Cary Franklin. “Setting the Pension Funding Target: Why Expected Return Matters.” *Benefits Magazine*, November 2013.

Society of Actuaries a. “Analysis of Target Benefit Plan Design Options.” February 2016. https://www.soa.org/resources/research-reports/2016/2016-target-benefit-plans/

Society of Actuaries b. “Target Benefit Plans Issue Brief #1: Exploring the Simplest Design.” 2016. https://www.soa.org/resources/research-reports/2016/2016-target-benefit-plans/

Society of Actuaries c. “Target Benefit Plans Issue Brief #2: Enhancing Security and Stability.” 2016. https://www.soa.org/resources/research-reports/2016/2016-target-benefit-plans/

Turner, John A, Humberto Godinez-Olivares, David D McCarthy, and Carmen Boado Penas. “Determining Discount Rates Required to Fund Defined Benefit Plans.” Oslo: IAA Colloquium, 2015.

Appendix A – Summary of Simulation Outputs

For each plan simulation, a summary of each evaluation measure described in section 4.5 was presented in a graphical or table form, as shown below.

A.1 Plan A Simulation

Membership profile: The evolution of membership throughout the simulation period is shown below.

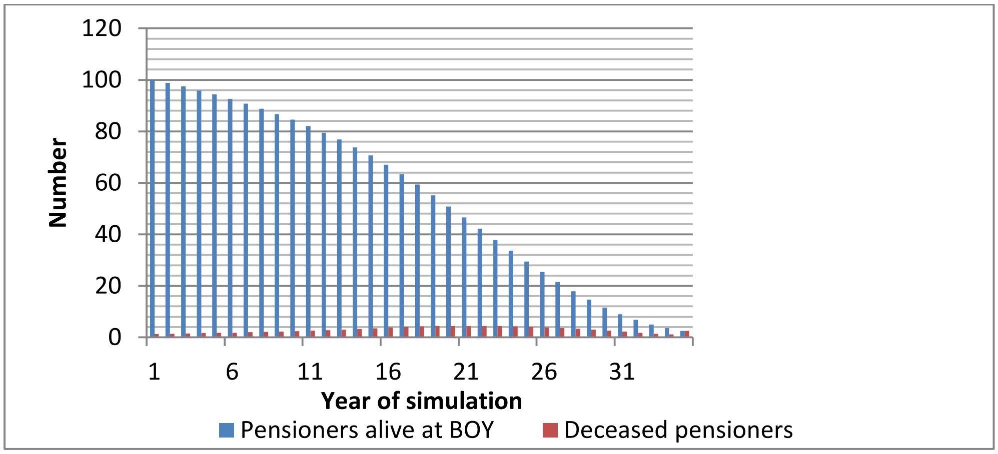


Figure 3: Distribution of pensioners

Measure 1: Distribution of average pension paid over the lifetime of each pensioner, by age at death

The average pension received over the lifetime of a pensioner is calculated using equation (6). The figure below shows the distribution (5th, 25th, 50th, 75th, and 95th percentiles) of average pension by age at death, for the two liability measurements.

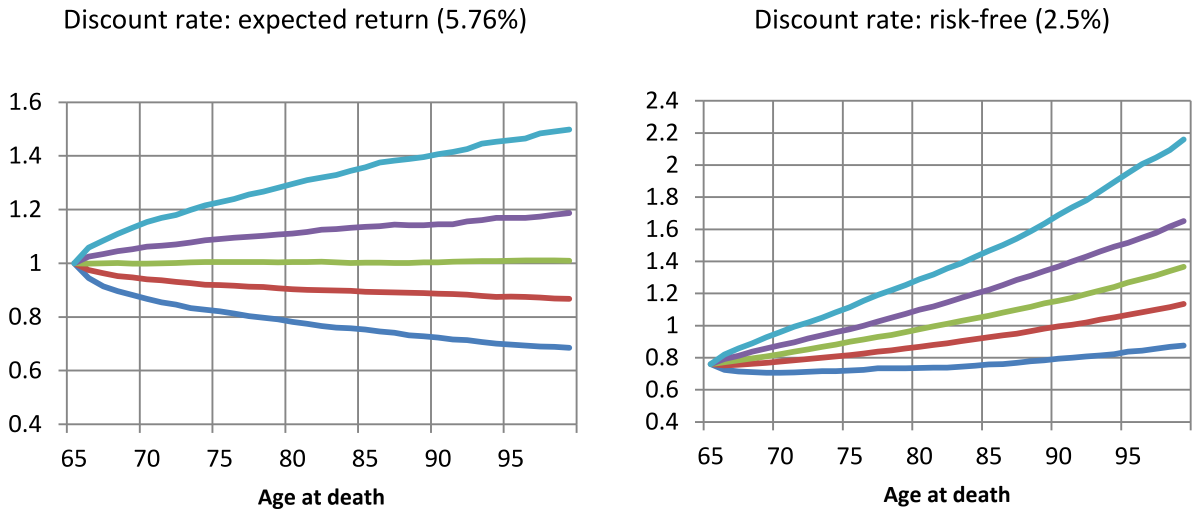
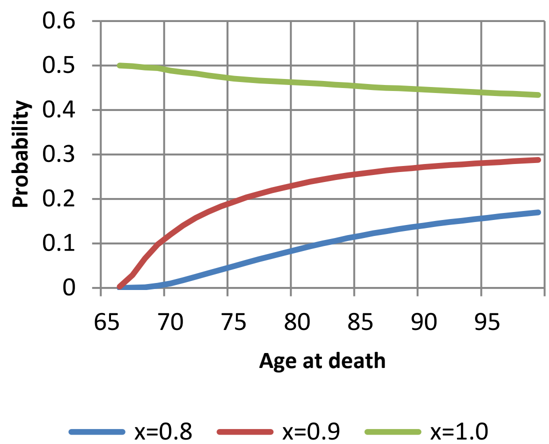


Figure 4: Distribution of average pension, by age at death

Measure 2: Probability of average pension falling below 1.0, 0.9, and 0.8, by age at death

The next figure shows the probabilities of average pension falling below thresholds $x = 1.0, 0.9$, and 0.8, by age at death, for the two liability measurements.

Discount rate: expected return (5.76%)



Discount rate: risk-free (2.5%)

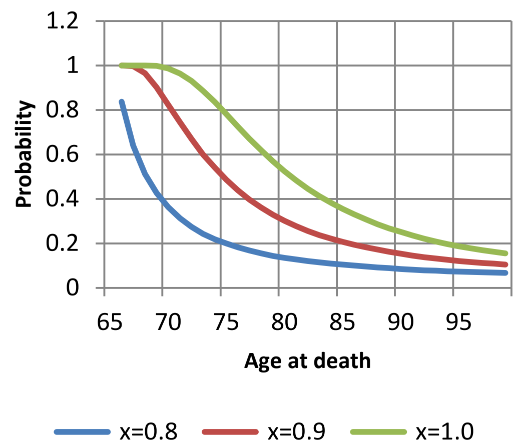


Figure 5: Probability of average pension falling below 1.0, 0.9, and 0.8, by age at death

Measure 3: Mean, median, standard deviation, skewness, and kurtosis for the distribution of average pension

Table 6 below provides key statistics for the distribution of average pension for the following ages at death: 70, 75, 80, and 90, separately for the two liability measurements.

Table 6: Key statistics for distribution of average pension

Discount rate: expected return (5.76%)

Age at death	**70**	**75**	**80**	**90**
Mean	1.00	1.01	1.02	1.03
Median	1.00	1.00	1.00	1.00
Standard deviation	0.09	0.13	0.16	0.21
Skewness	0.28	0.41	0.51	0.74
Kurtosis	-0.01	0.46	0.72	1.08

Discount rate: risk-free (2.5%)

Age at death	**70**	**75**	**80**	**90**
Mean	0.83	0.90	0.99	1.19
Median	0.82	0.90	0.98	1.15
Standard deviation	0.08	0.12	0.17	0.28
Skewness	0.30	0.46	0.58	0.80
Kurtosis	0.18	0.51	0.79	1.27

A.2 Plan B Simulation

Membership profile: The evolution of membership throughout the simulation period is shown below. After all pensioners in the first cohort are deceased, the number of pensioners (1,916) and their distribution by age will both become static.

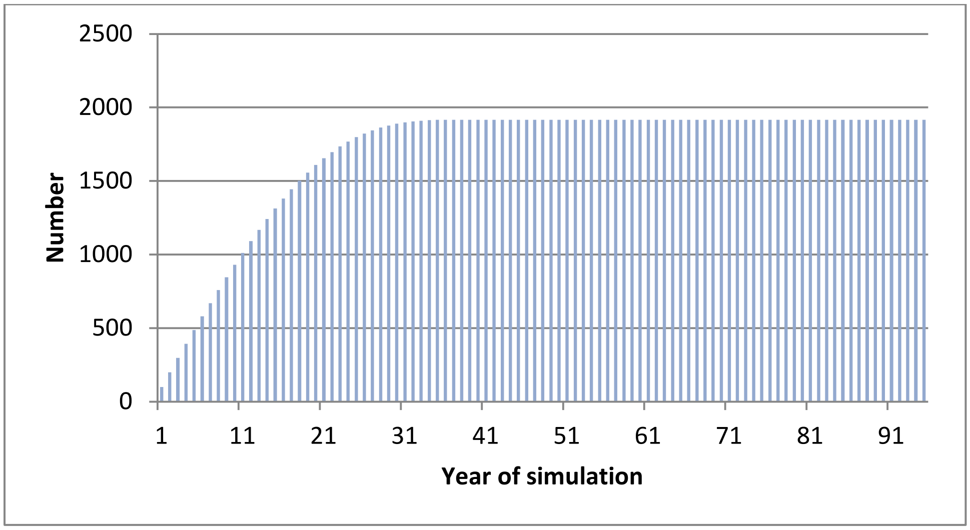


Figure 6: Membership profile

Measure 1: Distribution of average pension paid over the lifetime of pensioners, by cohort

The average pension received over the lifetime of pensioners in a cohort is calculated using equation (9). Figure 7 shows the distribution (5th, 25th, 50th, 75th, and 95th percentiles) of average pension by cohort of pensioners, for the two liability measurements.

Discount rate: expected return (5.76%) | Discount rate: risk-free (2.5%)

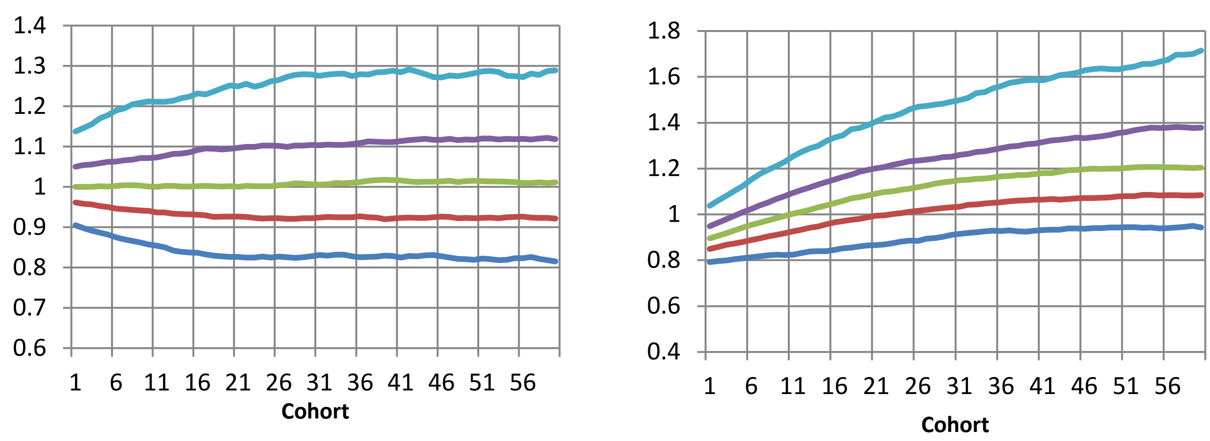
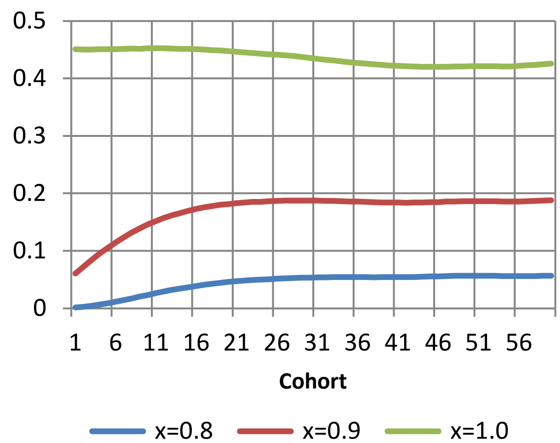


Figure 7: Distribution of average pension, by cohort

Measure 2: Probability of average pension falling below 1.0, 0.9, and 0.8, by cohort

Figure 8 shows the probabilities of average pension falling below thresholds $x = 1.0, 0.9$, and 0.8, by cohort, for the two liability measurements.

Discount rate: expected return (5.76%)



Discount rate: risk-free (2.5%)

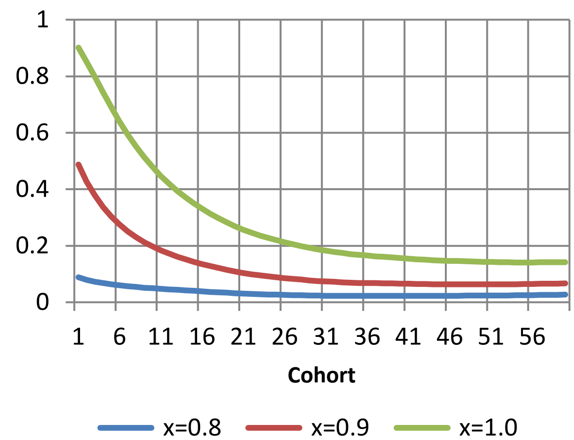


Figure 8: Probability of average pension falling below 1.0, 0.9, and 0.8, by cohort

Measure 3: Mean, median, standard deviation, skewness, and kurtosis for the distribution of average pension

Table 7 below provides key statistics for the distribution of average pension for selected cohorts: 1st, 10th, 25th, and 50th, separately for the two liability measurements.

Table 7: Key statistics for distribution of average pension

Discount rate: expected return (5.76%)

Cohort	**1**	**10**	**25**	**50**
Mean	1.01	1.01	1.02	1.03
Median	1.00	1.00	1.00	1.01
Standard deviation	0.07	0.11	0.13	0.14
Skewness	0.51	0.62	0.65	0.66
Kurtosis	0.41	0.62	0.68	0.93

Discount rate: risk-free (2.5%)

Cohort	**1**	**10**	**25**	**50**
Mean	0.90	1.01	1.13	1.23
Median	0.89	0.99	1.11	1.20
Standard deviation	0.08	0.13	0.17	0.22
Skewness	0.57	0.68	0.74	0.95
Kurtosis	0.46	0.74	0.88	2.00

Measure 4: Distribution of pension fund balance, by year of simulation

Figure 9 below shows the growth of pension fund balance over time.

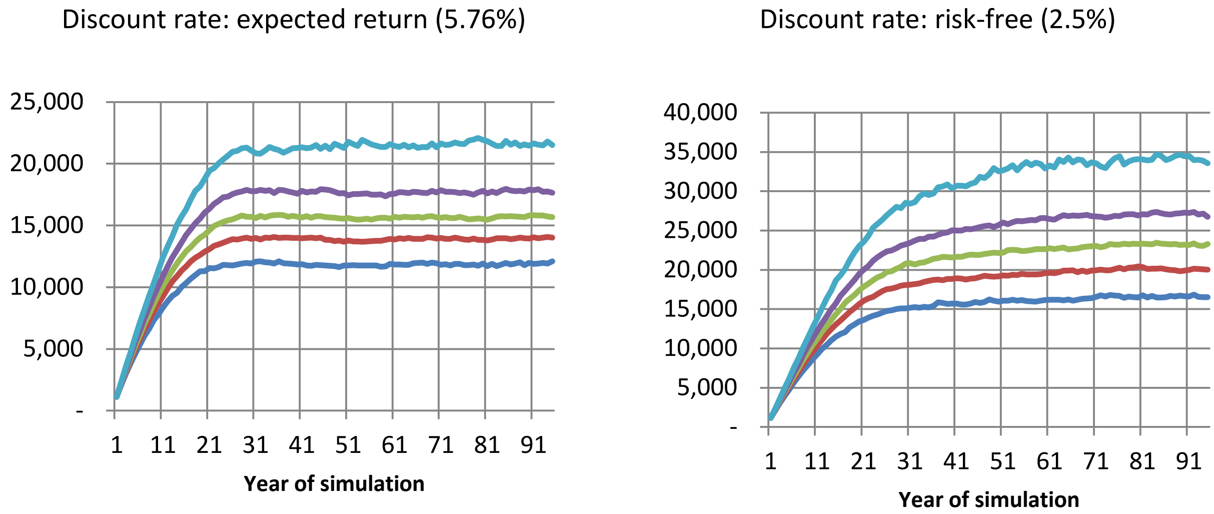


Figure 9: Distribution of pension fund balance, by year of simulation

A.3 Plan C Simulation

Membership profile: The evolution of membership throughout the simulation period is shown below. After the first cohort of members retires at the end of 35 years, the membership will become stationery with 3,500 members evenly distributed between age 30 and age 64 inclusive.

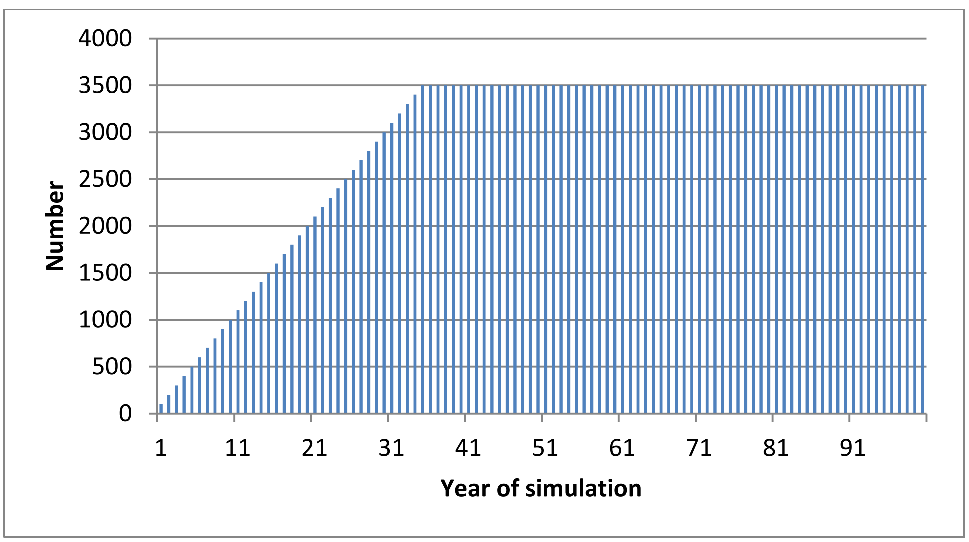


Figure 10: Membership profile

Measure 1: Distribution of retirement benefit paid to member, by cohort

The retirement benefit paid to the $(k+1)^{th}$ cohort, $k \geq 0$, at retirement is equal to B_{k+34}, which is calculated using equation (18). The figure below shows the distribution (5th, 25th, 50th, 75th, and 95th percentiles) of retirement benefit by cohort of members, for the two liability measurements.

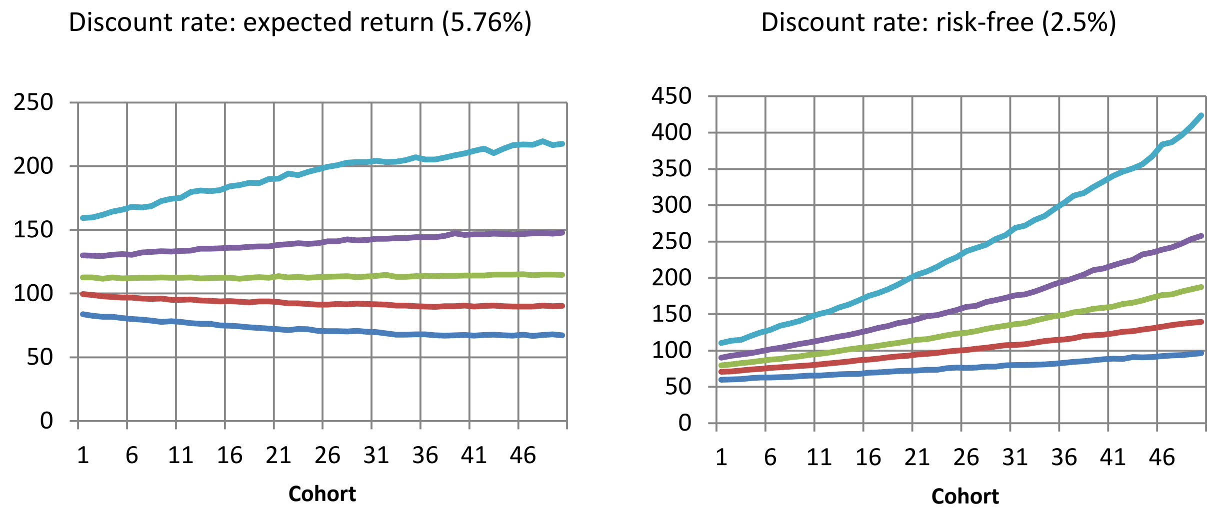
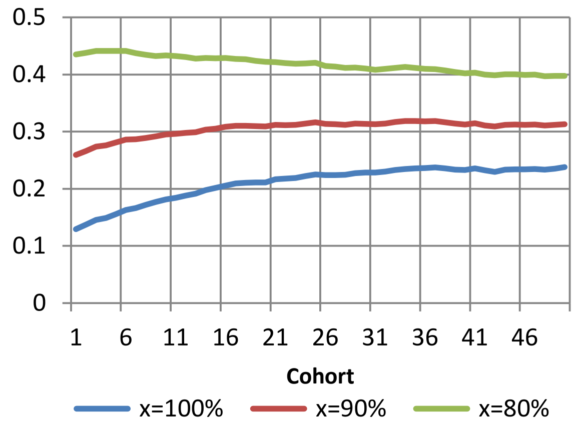


Figure 11: Distribution of retirement benefit, by cohort

Measure 2: Probability of retirement benefit falling below 100%, 90%, and 80% of target retirement benefit, by cohort

The target retirement benefit is set as 112 (section 4.3). The following figure shows the probabilities of retirement benefit falling below 100%, 90%, and 80% of the target retirement benefit, by cohort, for the two liability measurements.

Discount rate: expected return (5.76%)



Discount rate: risk-free (2.5%)

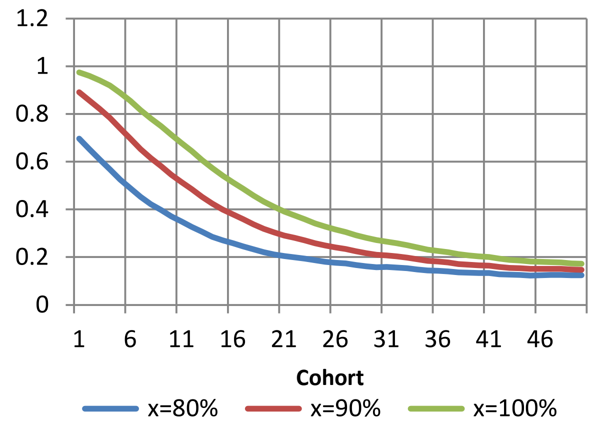


Figure 12: Probability of retirement benefit falling below 100%, 90%, and 80% of target retirement benefit, by cohort

Measure 3: Mean, median, standard deviation, skewness, and kurtosis for the distribution of retirement benefit

Table 8 below provides key statistics for the distribution of retirement benefit for selected cohorts of members: 1st, 10th, 25th, and 50th, separately for the two liability measurements.

Table 8: Key statistics for distribution of retirement benefit

Discount rate: expected return (5.76%)

Cohort	**1**	**10**	**25**	**50**
Mean	115.9	117.2	120.3	125.0
Median	112.6	112.3	112.9	114.5
Standard deviation	23.2	30.2	40.4	49.6
Skewness	0.75	1.07	1.48	1.49
Kurtosis	0.76	1.95	4.51	3.21

Discount rate: risk-free (2.5%)

Cohort	**1**	**10**	**25**	**50**
Mean	81.6	98.0	133.5	213.4
Median	79.4	94.2	123.0	187.3
Standard deviation	15.7	25.2	48.0	107.3
Skewness	0.91	1.09	1.33	1.83
Kurtosis	1.48	1.84	2.27	5.35