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# Population Aging, Implications for Asset Values, and Impact for Pension Plans: An International Study Phase 2 Final Report



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# Population Aging, Implications for Asset Values, and Impact for Pension Plans: An International Study

Phase 2 Final Report

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# Chapter 1

## Introduction

In 2015 an international team of researchers at the University of Waterloo (Canada) and the University of Kent (United Kingdom) began a multi-disciplinary research project entitled “Population Aging, Implications for Asset Values, and Impact for Pension Plans: An International Study”. The project was funded initially by a Partnership Development Grant from the Social Sciences Humanities Research Council (SSHRC) and initial partners, the Society of Actuaries (SOA) and the Institute and Faculty of Actuaries (IFoA). Subsequently, the Canadian Institute of Actuaries (CIA) agreed to be a partner and provide some funding. In addition to funding, all three actuarial organizations have appointed members to a Project Oversight Group (POG) that has provided advice throughout the project. The overall project is scheduled for completion in 2019.

There are many stages along the way to the completion of a multi-year project. This report completes one such stage. The work presented in this report has two main components:

- A literature review regarding the demographic impact on the returns of various asset classes; and

- A model specification to analyze the impact of demographics on house prices.

This report is organized as follows. Chapter 2 summarizes the literature for each of the following asset classes: equities, bonds, housing, and infrastructure. The structure of the chapter is explained at its beginning, and a conclusion regarding the literature review ends the chapter. A draft version of the literature review was made available to members of the actuarial organizations in mid-2017, for their use and information, and to provide members with an opportunity to comment. The literature review in this report contains several additional papers published since the draft literature review was posted.

In Chapter 3 we propose an extension to models developed by Takats [71] and Nishimura & Takats [59] that are based on a relationship between demographic factors and house prices. Their models assume that land is in fixed supply, which we relax. This relaxation seems more realistic for countries such as Canada that have a large land mass and a comparatively small population. The model is specified and we discuss how we plan to test it empirically, but the actual testing of the model is beyond the scope of this stage of the project.

In Chapter 4 we acknowledge the work, ideas, and funding provided by many individuals and organizations. The papers cited are listed in the Bibliography at the end of this report.

## Chapter 2

# Literature Review Regarding the Connection between Demographic Factors and Asset Class Returns/Prices

### 2.1 Introduction

The main purpose of this chapter is to identify and summarize academic and other papers that draw a connection between population structure and asset class returns (or prices). Specific asset classes of interest are equities, bonds, housing, and infrastructure.

Our methodology to find the appropriate papers is as follows.

- We searched for papers published since 2000 whose titles contained some link between demographic factors and the asset classes of interest. Of note, papers that study generic “risky” and “risk free” assets are not included.



- Using the papers initially identified, we examined references in those papers that satisfy the initial criteria.
- Using “Google Scholar”, we checked the papers that have cited the articles already identified to see if they satisfy the initial criteria.
- Based on the papers collected so far, we identified three key papers published before 2000 for inclusion in the review.
- Finally, since the methodology so far had not identified any papers that consider infrastructure, we searched for papers published since 2000 whose titles contained some mention of infrastructure.

This methodology surfaced over 60 papers in total, of which two key papers address stock and bond returns. Fama & French [24] provide empirical analysis of stock and bond returns, but with no connection to demographic change. Bakshi & Chen [6] show a connection between demographic shifts and asset returns. Mankiw & Weil [52] link returns on housing to demographic shifts.

In the rest of this chapter, we first provide some background on the factor pricing approach, and the role of the Fama & French<sup>1</sup> point of view is introduced. In the next section, we group various demographic factors that have been used as independent variables. Then, we have a number of sections that focus on each asset class. Finally, Section 2.8 concludes.

## 2.2 Background

Fama & French (1992) [23] and Fama & French (1993) [24] find evidence that variables other than “the market”<sup>2</sup> are better able to explain observed

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<sup>1</sup> [23], [24], and [25].

<sup>2</sup>The market is the sole factor in the CAPM model.

returns. In particular, Fama & French (1993) [24] find that two empirically determined variables, size<sup>3</sup> and book-to-market ratio<sup>4</sup> provide a reasonable explanation of the cross-section of average returns on NYSE, Amex, NASDAQ stocks for the 1963-1990 period. These two factors explain the differences in average returns across stocks, though they also use a broad market factor in order to explain why stock returns are on average above the one-month bill rate.

The work of Fama & French has evolved since the initial papers in 1992 and 1993. Their current approach, set out in Fama & French (2016) [26] and Fama & French (2017) [27], adds two more factors to improve the model. These two factors are a measure of profitability and a measure of investment (i.e., reinvesting money in the business). Fama & French find that these two factors improve the fit of the model to the data.

For bonds they continue to use a two-factor model. The first factor, TERM<sup>5</sup>, is a proxy for the common risk in bond returns that arises from unexpected changes in the short-term interest rate. The second factor, DEF<sup>6</sup>, is a proxy for the shifts in economic conditions that change the likelihood of default. Analytically, the bond-market factors TERM and DEF prove to be strong predictors of bond returns.

Bakshi & Chen [6] is the first major paper to consider the linkage between demographic shifts and asset returns. Their premise, from the life-cycle investment hypothesis [58], suggests that the stock market should rise, but the housing market should decline with an increase in the average age of the

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<sup>3</sup>Stock price times number of shares.

<sup>4</sup>The ratio of the book value of a firms common stock to its market value

<sup>5</sup>TERM is the difference between the monthly long-term government bond return and the one month treasury bill rate measured at the end of the previous month.

<sup>6</sup>DEF is the difference between the return on a market portfolio of long-term corporate bonds and the long-term government bond return.

population. This prediction is supported in the post-1945 period in the US. A second hypothesis that they test is that an investor's risk aversion should increase with age. They find support for this hypothesis in the post-1945 period in the US, where an increase in the average age predicted a rise in risk premia.

## **2.3 Demographic Factors Used as Independent Variables**

Demographic studies have analyzed a number of factors since they started in a modern sense in the 17th century by John Graunt. Demographic trends are by their nature very smooth. They do not contribute to short-run noise, but are a natural candidate to capture the information that emerges in the long-run. To capture the overall impact of a population's change, three factors are most important: fertility, mortality, and migration.

In this section, we categorize the demographic factors that have been used as independent variables in asset pricing studies. They are divided into three distinct categories: specific age intervals, working-age related factors, and other factors.

### **2.3.1 Age Intervals and Their Related Ratios**

Although a majority of researchers use a set of common age intervals and ratios from previous work in order to describe linguistic terms (e.g., young, old, middle age), other researchers prefer to define and apply their own definitions. What follows is a list of definitions that have been used in the papers included in this literature review.

- Sets of three linguistic terms (young, middle age, and old age) and their related ratios (e.g., population aged 20-39 as a percent of the

total, population aged 40-64 as a percent of the total, and population 65+ as a percent of the total);

- Sets of four linguistic terms and their related ratios. Typically, people aged 0-14, people aged 15-39, people aged 39-64 years, and people aged 65+ are named young, low middle age, high middle age, and old age, respectively;
- Sets of seven age groups using 10-year intervals ( $< 20$ ; 20-29; 30-39; 40-49; 50-59; 60-69; and  $> 70$ , and their related ratios);
- Sets of 15 age groups using 5-year intervals (0-4,  $\dots$ , 70+);
- A set of 81 single-year age groups (ages 0, 1,  $\dots$ , 79, 80+) or the number of people of a particular age in a particular year; and
- Specific ratios like the “Yuppie-Nerd” ratio (the ratio of the number of 20-34 year olds to the number of 40-54 year olds) and the fraction of the total population between ages 25 and 35.

### 2.3.2 Working-Age Factors

This type of factor is important because it plays a key role in several economic theories. The main factors of this type are the following:

- The ratio of the Working-Age population (persons aged 15-64 or 25-64) to the total population;
- The Working-Age population as the entire population between 25 and 64 years of age;
- The Population in the Prime Earnings Ages (45-64); and
- The Old Age Dependency ratio, defined as the ratio of the population aged 65+ to the working-age population (ages 20-64).

### 2.3.3 Other Demographic Factors

The factors of this type that are seen most frequently are the following:

- Total population;
- Population growth rate;
- Mortality, mortality rate of adults over 25 years old, and mortality shock;
- Annual number of live births;
- Median age;
- Average normalized age;
- Life Expectancy (in years);
- Average age of the adult population;
- Gender in retired couples (husbands and wives); and
- Other factors, such as number divorced, single, or female.

## 2.4 The Stock Market and Demographic Factors

In this section, we classify all the research that considers the impact of demographic factors on some aspect of the stock market. In this section, we divide the dependent variables used into four distinct categories – stock price indices, price-earnings ratio, yield and returns, and market structure. We separate these categories by the demographic factors that are mentioned in Section 2.3 as independent variables. Table 2.1 provides a summary.

### 2.4.1 Stock Price Indexes

Stock price indices have played a vital role in stock market analysis. However, some researchers prefer to make a variety of adjustments in order to focus on different issues. The results generally show that stock prices (or returns) are positively affected by an increase in the proportion of the “middle-aged” population and negatively affected by an increase in the proportion of the “old age” population.

- The US Government Accountability Office [10] concludes that there should be little impact of retiring baby boomers on US stock prices after controlling for economic variables.
- Ang & Maddaloni [3] use data from the G5 countries and show that an increase in the proportion of the population over age 65 is associated with a reduction in the equity risk premium.
- Maurer [56] creates a theoretical model (calibrated to the US) that suggests that higher fertility and/or greater longevity result in a higher equity risk premium. Similarly, lower fertility and/or lesser longevity result in a lower equity risk premium.
- Davis & Li [20] use data from seven large OECD countries and conclude that real stock prices are positively affected by the proportion of the population aged 20 - 39 and 40 - 64, increasing prices by 2% to 3% for each 1% increase in either of these population proportions.
- Huynh et al. [39] use Australian data and find that an increase in the share of the population aged 40 - 64 positively affects share prices.
- Lim & Weil [50] consider the change in the labour force participation rate in the US. They conclude that stock prices will increase by 8% starting in the 1990's and peaking around 2012. There will then be a decline of 16.5% over the following two decades.

- Kedar-Levy [46] also uses the labour force participation rate in the US as an independent variable. Unlike Lim & Weil [50], he finds that the projected decrease in this rate starting in 2016 should result in an *increase* in stock prices by 0.22% per year relative to the situation of no change in the rate.
- Brooks [11] looks at the impact of age shares. Using data from 16 developed countries, the findings show that an increase in the share of the population under age 35 depresses stock prices, as does an increase in the share of the population age 65 and older. An increase in the share of the population from age 60 - 64 increases stock prices. While there is some variation across countries, these general results hold, regardless of country. Park [60] uses G5 data and finds similar results; while Bae [5] uses US data and also finds similar results.
- Jianakoplos & Bernasek [45] study both age and cohort effects on risky asset allocation. They find that the portfolio allocation of assets to equities (and bonds) reduces with age, and somewhat surprisingly, with younger cohorts. This suggests a depressing effect on share prices of an aging population.

## 2.4.2 Price-Earnings Ratio

Geanakoplos et al. [34] derive some predictions from their model, which they then compare with historical data on US stock market. The first prediction is that price-earnings (PE) ratios should move proportionally with the ratio of middle-aged to young adults. Secondly, they show in their model that the equity premium should covary with the ratio of middle-aged to young adults, even though the young are more risk-tolerant than the middle-aged. Due to changes in this demographic ratio, their model predicts a decline in the PE ratio in the US equity market over the next twenty years. They also mention that the only real prospect for offsetting the effect of a small generation

of middle-aged agents buying the equity of a large retired generation comes from increased participation in the US securities market by investors from the developing countries.

Jamal & Quayes [44] use the price of S&P 500 stocks, normalized for dividends (price - dividend ratio) as the dependent variable and the proportion of population in the Prime Earning Ages as demographic factor in their regression model. They claim that each percent change in the proportion in this age group results in approximately a 5% change in the price of stocks, normalized for dividends. The US Census Bureau projects the proportion of the population in the 40-64 age group to decline from 30.4% in 2000, to 28% in 2030. This represents a 7.9% drop in the ratio of this prime savings group as a proportion of the total population. Their analysis suggests this would lead to approximately a 39% decline in the price - dividend ratio.

Using updated data, Quayes & Jamal [65] find slightly different results. They find that a 1% increase in the proportion of the population in the Prime Earning Ages results in approximately a 1.0% increase in the price of stocks, normalized for dividends. While a 1% change in the proportion of the population over age 65 results in a 2.3% reduction in the price of stocks, normalized for dividends. Looking at these two papers together calls into question the effectiveness of using population proportions as an explanatory variable of the magnitude of change in the price - dividend ratio.

Roy et al. [68] claim that the link between stock prices and demographic trends is impacted by the life cycle theory of asset accumulation/decumulation and portfolio choice. Asset accumulation and asset decumulation patterns today are very different from the past. They suggest the need to redefine age ranges used traditionally to explain asset prices and economic variables in the future. They construct and compute a ratio of the middle-aged population



to old-age population, known as the “Middle/Old” ratio to explain the price-earnings ratio of the stock index. For the US, based on the results from the Survey of Consumer Finance, the Middle/Old ratio and the real S&P500 P/E ratio have a strong correlation, at 0.73, during the period 1950-2011. Using projected data for the Middle/Old ratio, they forecast a declining P/E ratio for the US from 16.1 currently to 5.2 in 2025. Moreover, they find that the relationships between stock P/E ratios and the Middle/Old ratio for France, Germany and Japan are weak because the appropriate age ranges that should be used to define the Middle/Old ratios might be different for these countries.

Ratanabanchuen [66] shows that the gradual increase in the population aged 35-40 since the 1970s has led to a significantly positive movement in the real prices of common stocks. Moreover, this paper shows a statistically significant relationship between the 5-year percentage changes in the fraction of the 35-49 age groups and 5-year real equity returns in the UK.

### 2.4.3 Yield and Returns

Although earnings yield<sup>7</sup> is defined as EPS divided by the stock price (E/P) and it is the reciprocal of the P/E ratio, we have placed it in a separate category due to its ease of comparing potential returns across different types of assets.

Poterba [63] examines the impact of demographic changes on dividend yield. His results show that the population share between the ages of 40 and 64 has a positive and significant correlation with the Price/Dividend ratio. The population share over age 65 also has a positive correlation, although the magnitude is about half that of the population share between the ages

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<sup>7</sup>Researchers in finance typically use dividend/price ratio, where the study of the variability of this ratio is of interest. Whereas, investors typically emphasize the yield or return that they will get from a particular stock.

of 40 and 64.

Marekwica et al. [53] analyze the relationship between demographic structure and real stock returns for the G7 countries. They use two ratios - population aged 20 to 39 divided by population aged 65 and older (the “young” ratio), and population aged 40 to 64 divided by population aged 65 and older (“middle age” ratio). In order to address the issue of integrated international capital markets, the demographic factors are not only calculated separately for each country under consideration, but also for all G7 countries as a whole. They claim that there was no robust relationship found between shocks in demographic variables and asset returns in the framework of these models. (Significant results can only be obtained for Canada, where the coefficient for the first ratio does not have the expected sign, while the second ratio is positive.)

The long-run analysis of Favero et al. [30] shows that there is a stable cointegrating vector between the dividend-price ratio, total factor productivity, and a demographic factor (the ratio of the number of people aged 40-49 to the number of people aged 20-29). Based on their results, they exploit the exogeneity and predictability of demographic factors to simulate the equity risk premium to 2050. Their results point to some, though not dramatic, decline of the equity risk premium for the next 10 years.

Brunetti & Torricelli [12] believe that the dynamic pattern of the aging phenomenon plays a fundamental role in determining the explanatory power of demographic variables in empirical studies. In particular, their regression results suggest that an increase in the proportion of early working-aged (20 to 39) tends to have a positive effect on the returns on stocks, while the opposite is true for the late working-aged (40 - 64).

Favero et al. [29] predict a positive correlation between the proportion of middle-age to young population ratio (proportion of the number of people aged 40-49 to the number of people aged 20-29) and market prices, consequently a negative correlation with the dividend yield.

Favero & Tamoni [32] augment the set of predictors of stock market returns by considering the dividend-price ratio together with a demographic variable, the ratio of middle aged (40-49) to young (20-29) population, that captures the slow moving but time-varying mean of the US dividend-price ratio. Based on their research, the co-movement between demographics and stock market returns is negligible for annual returns and remarkable for 20-year returns. Importantly, they claim that the slope of the term structure of the stock market estimated within their proposed framework remains downward sloping even when the two additional sources of risk proposed by Pastor & Stambaugh<sup>8</sup> [62] (who extend the framework of Campbell & Viceira<sup>9</sup> [13]) are considered.

Lee [49] finds that, as consistent with the behavioral life cycle hypothesis, the long-run returns of dividend-yield strategies are positively driven by demographic clientele variation as represented by changes in the proportion of the older population, defined as the proportion of population aged above 65 to the total population. In essence, he claims that the larger the increase in the proportion of the old population, the greater the relative demand for

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<sup>8</sup>Pastor & Stambaugh in 2008 [62] extend the framework of Campbell & Viceira [13] to consider two additional sources of risk. One source reflects the uncertainty around the mean of the process generating returns and the other source reflects the uncertainty of the parameters. Within this extended framework, Pastor & Stambaugh [62] do find a positive slope for the term structure of stock market risk, despite the evidence of predictability

<sup>9</sup>Campbell & Viceira [13] in 2005 and more recently Schotman et al. [70] in 2008 have shown that whereas absence of predictability entails a flat term structure of risk, predictability per se does not lead to a downward sloping term structure of risk. In fact, risk at the different horizons is determined by three components: i.i.d uncertainty, mean reversion, and uncertainty about future predictors.

high dividend-paying assets, hence the stronger the relative performance of dividend-yield investment strategies.

Using a large sample of countries and 60 years of data, Arnott & Chaves [4] find a strong and intuitive link between demographic transitions and both GDP growth and capital market returns. They measured stock and bond returns as excess returns relative to domestic cash returns, rather than as simple annualized returns. They find that large populations of retirees (65+) seem to erode the performance of financial markets as well as economic growth. They claim that Japan, Finland, and Sweden have a dangerous combination of very low birth rates and an exploding number of retirees, giving them a strong demographic headwind. Their results for Canada, the United States, and central Europe are mixed, with slightly negative or positive projections, depending on the measure used.

Instead of focusing on population changes among various age groups as in the previous literature, Huang et al. [38] adopt a different approach by directly examining the relationship between an unexpected shock to the mortality rate and the equity premium. Since changes in demographic structure are determined by several factors, such as the current demographic structure, the mortality rate, the fertility rate, and immigration policies, separating the impact of the mortality rate from the impacts of other factors could provide a more accurate measure of the sensitivity of the equity premium with respect to the mortality rate. It could also reduce the basis risk faced by financial institutions when hedging mortality-linked securities. By using a one-year horizon, they find that the magnitude of the increase in the risk premium is about 0.54% per year when the mortality rate is one basis point lower than expected. They also find similar results when testing the long-run relationship. As can be seen, their empirical evidence finds strong support for the view that an unexpected shock to the mortality rate is significantly nega-

tively correlated with the risk premiums.

Kedar-Levy [47] projects demographic effects on equity returns between 2010 and 2050. His model is calibrated for postwar parameters of US financial markets, as well as the 2004 Census Bureau population aging forecast until 2050. Based on the benchmark estimate of an average normalized life expectancy<sup>10</sup>, the result shows how normalized age increases (and hence the horizon declines) from 0.439 in 2010 to 0.469 in 2050. As a result of aging, the average annual decline in equilibrium return over the entire period 2010 - 2050 is estimated to be 185 basis points. This finding is higher than the 60 - 100 basis points per annum decline estimated by Geanakoplos et al. [34] in 2004.

#### 2.4.4 Market Structure

It is difficult to find the real impact of the population structure on asset prices due to the fact that the supply of assets should adjust rapidly to changes in asset prices. If the asset markets are efficient and the supply of assets is very elastic to rise of asset prices, then the rise of asset prices would induce an immediate increase in the supply of assets, thus not reflecting any real change in asset prices (Park & Rhee [61]). Therefore, some researchers have been working to analyze the relationship between population structure and some index that represents the size of the asset markets.

Goyal [35] is the first one who considers stock market out-flows as a dependent variable. He finds that stock market out-flows are positively correlated with the age 65+ proportion of the population and negatively correlated with the prime working age (45-64) proportion of the population. Further-

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<sup>10</sup>Average normalized life expectancy is the proportion of total expected lifetime in the future. Similarly, average normalized age is the proportion of total expected lifetime lived to date.

more, population structure adds explanatory power to equity risk premium regressions. Population structure also adds explanatory power to the investment/savings rate for the US economy. Finally, international demographic changes have some power in explaining international capital flows.

Davis [19] conducts empirical work based on the experience of 72 countries (of which 23 are OECD countries, 36 are emerging market economies, and 13 are transition economies). Viewed in the light of the existing literature, he suggests that demographic changes have had a detectable impact on financial structure. He finds a significant positive effect of the proportion of those aged 40-64 on equities, consistent with risk aversion effects.

Park & Rhee [61] consider the size of stock markets, which they measure as the aggregate value of listed stocks as a proportion of GDP, as the independent variable. They claim, however, that population composition variables such as the proportion of population aged 40-64 and the proportion aged 65 and over do not show a strong relationship with real stock returns. The proportion of population aged 65 and over has a negative relationship with real stock returns. The finding that the population between the age 40 and 64, who have a higher tendency to invest in stock and own greater financial wealth, does not correlate with real stock returns agrees with Poterba [64]. In one of their models in which only the population composition was considered as the explanatory variable, an increase in the proportion of elderly population (+65) considerably increases the size of stock markets.

### **2.4.5 Other considerations**

Some other researchers analyze the impact of demographic changes on household portfolio choice. A few of them do not consider stocks and bonds separately. Fagereng et al. [21] claim that the portfolio share in risky assets is high and constant in the earlier and mid phases of the life cycle at a level just

below 50%. They also point out that as retirement approaches, households start re-balancing their risky asset share gradually but continuously at a pace of a little less than one percentage point per year until they retire (around age 65). In retirement, investors who remain in the stock market keep the share flat at around 30%.

Mayordomo et al. [57] show that in the case of age, the optimal maximum investment in stocks should be by households headed by a person between the ages of 45 and 55 years, and decreasing in subsequent age groups. This result is not consistent with the actual data, which shows a linear relationship between stock ownership and age, such that the highest investment in stocks is by the group older than 55 years.

Kraft & Munk [48] claim that the fraction of total wealth invested in stock consists of a constant position of 23% - 28%, depending on the level of human capital (education and training).

Rausch [67] applies an OLG model with heterogeneous households. He claims that a decline in fertility rates increases the per capita capital stock, consumption, and output, and decreases investment. Population aging leads to a higher per capita capital stock during the transition and in the new long-run equilibrium.

Finally, Addoum [1] shows that couples significantly decrease their stock allocations after retirement, whereas singles' allocations remain relatively unchanged.

## **2.4.6 Summary of Equities**

At the end of this section, it is worth mentioning that these researchers use various econometric techniques. Some of them concentrate on a specific

country and some of them work on a group of countries. Table 2.2 provides a summary of such information regarding the articles covered in this section.

## 2.5 The Bond Market and Demographic Factors

The notion that there should be a demographic influence on bond returns is often based on the Life Cycle Savings hypothesis, proposed by Modigliani & Brumberg [58] in the early 1950s. In short, this hypothesis suggests that lifetime utility is maximized when people borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired.

Kraft & Munk [48] claim that the optimal allocation to bonds is quite high while young, remaining constant for a while before dropping rapidly to virtually nil at the point of retirement. Their results are affected by using a 20-year zero-coupon bond as the bond instrument.

Hassan [37] conducts a survey of articles that investigate whether age structure affects asset markets and the real exchange rate. Their survey is clustered into Single Country Studies and Cross-Country Studies. A key part of their survey is the observation from the literature that age structure affects the real exchange rate.

In this section, we divide the literature into three categories by dependent variable: bond yields, bond prices, and short-term interest rates. Within each category, the articles are divided by type of demographic variable, as described in Section 2.3. Table 2.3 provides a summary.



### 2.5.1 Bond Yields

Davis & Li [20] use three demographic intervals as independent variables in order to model the real bond yield in seven OECD countries over the past 50 years. The 40-64 age coefficients are significant and negative (reducing yield) in all countries. Of the 20-39 age coefficients, only four are significant (all are positive, increasing yield), the US, UK, France, and Italy.

Poterba [63] claims that the most substantial correlation is between demographic factors and T-Bill returns. He surmises that this is because there is less variation in the T-Bill returns than either equity or bond returns, allowing the relationship with the demographic variables to be detected. Moreover, based on his model fitted to US data from 1926 - 2003, he finds an abnormally large expectation of falling real bill yields (650 basis points) and falling real bond yields (900 basis points) between 2000 and 2040. He does not find this extent of drop plausible and suggests that omitted variable bias in his regressions may be causing this.

Marekwica et al. [53] analyze the interrelation between demographic factors and asset returns in the G7 countries to test the “Asset Meltdown” hypothesis empirically. They investigate the impact of various demographic variables on long-term government bond yields: the fraction of young and middle aged population in one regression model, and life expectancy and fertility in another model. However, their results for bonds and treasuries do not support the hypothesis of an Asset Meltdown.

Brunetti & Torricelli [12] regress government bond yields on demographic and/or economic control variables using annual data over 1958-2004. They find out that in Italy only the younger share, age 20-39, of the population actually affects the dynamics of the long-term Government bond yields, having an inverse relationship.

Arnott & Chaves [4] use dividend yields, three-month bill yields, and 10-year bond yields as control variables in their regressions. Their explanatory variables are the percentage of total population by age group and the change in these demographic shares. They estimate the joint effect of all demographic variables in the regressions through a polynomial model. One common theme emerges from all their analyses: large populations of retirees (65+) seem to erode the performance of financial markets as well as economic growth. This effect is less pronounced for bonds than equities, likely because they are sold later in retirement than stocks, based on widespread financial advice.

Favero et al. [31] develop a simple model of the yield curve. They consider the ratio of middle-aged (40-49) to young (20-29) population in the US as the relevant demographic variable to determine the persistent component of interest rates. They also consider the performance of augmenting autoregressive models for nominal bond yields and continuously compounded annual inflation against a benchmark where the effect of demographics is forced to be nil. Their model predicts a negative correlation between their demographic variable and bond yields.

Roy et al. [68] include both demographic and economic variables in their regressions. The result from regressing 10-year government bond yields on the Yuppie-Nerd ratio (the ratio of the number of 20-34 year olds to the number of 40-54 year olds) and inflation provides a good regression fit. Bond yields tend to go up when the Yuppie/Nerd ratio rises, and vice versa.

The regression results from Ratanabanchuen's models [66] show that changes in the fraction of the 45-49 and 60-64 age groups appear to have significant forecasting power for the changes in the yields of 3-year GILTs,

5-year GILTs, and 10-year GILTs. The coefficient estimates are all significantly negative indicating that a rising proportion of the 60-64 age group is associated with an increase in bond prices.

### **2.5.2 Bond Prices and Market Size**

Davis et al. [19] consider bond market capitalization as a proportion of GDP, as the dependent variable. They find a positive impact of the 65+ generation on the bond market size, consistent with the idea older people have greater risk aversion. There is a relative switch by the elderly from equities to bonds.

Brooks [11] investigates the total return bond index and total return Treasury Bill index as the dependent variables. His research suggests that real bond and Treasury Bill prices are negatively and significantly related to the middle-aged cohorts (between ages 40-44 and 60-64). While, real Treasury bill prices are positively related to the age 65+ variable. These findings indicate that older investors may shift their financial asset holdings towards Treasury bills in retirement in order to reduce consumption risk.

Park [61] shows that an increase in the population aged 65 and over increases the size of bond market. He claims that the positive correlation between the proportion of the elderly population and the size of bond market does not change if other explanatory variables are added.

### **2.5.3 Interest rates**

Maurer [56] regresses real interest rates, stock market excess returns and conditional volatilities on demographic variables using US data for the years 1926 to 2006. He uses two standard demographic indices, historical US birth and death rates, as the demographic regressors. The result suggests that in both the data and the model, the interest rate reduces as the birth rate

increases, and it increases as the death rate increases.

Ikeda & Saito [41] consider the real interest rate, computed as the one-year government bond yield minus the inflation rate of the GDP deflator. Their results imply that the reduction in the ratio of workers to the total population has worked to lower the real interest rate in Japan since the late 1980s, and this factor is expected to keep the interest rate low in the future. They claim that variations in this ratio are more important than changes in the total population itself. Their quantitative analysis indicates that growth in productivity is the dominant source of fluctuations in the real interest rate, but the demographic factor is also quantitatively important especially for its long-term movements.

Aksoy et al. [2] find a distinct life-cycle pattern with growth, investment and interest rates dropping as fertility increases, recovering as the proportion of working age population increases and falling again as the weight of old age dependents increases. Moreover, aging affects the savings decision of workers through those real interest rates. A permanent increase in longevity (increase in life expectancy) leads to increased growth rates in the short-term through the decrease in the workers' marginal propensity to consume. As a result, it leads to lower real interest rate and an increase in innovative activity.

Carvalho et al. [16] present a lower bound for the effects of the demographic transition on the real interest rate. In response to the demographic transition, they claim that the real interest rate should progressively fall from 4% to approximately 2.5% between 1990 and 2014. Their model explains about one third of the overall decline observed in the data. Furthermore, their simulation predicts that the real interest rate will fall an additional 50 basis points over the next forty years, before stabilizing around its new steady state value of 2%. The second important claim in their paper is that

the increase in the probability of surviving, rather than the fall in the population growth rate, is mainly responsible for the decline in the real interest rate caused by the demographic transition.

Favilukis & Sheng [33] provide novel empirical evidence from a cross-section of U.S. Metropolitan Statistical Areas (MSAs) that suggests that the relationship between demographics and interest rates is more complicated than previously thought. While older MSAs indeed have lower bank lending rates, they also have higher deposit rates. They build a model with imperfect competition in the banking sector that can rationalize some of these findings.

#### **2.5.4 Other considerations**

At the end of this section, it is worth mentioning that these researchers use various econometric techniques. Some of them concentrate on a specific country and some of them work on a group of countries. Table 2.4 provides a summary of such information regarding the articles covered in this section.

## **2.6 Housing**

Within this literature, Mankiw & Weil [52] is the seminal paper. They look at the age-specific housing demand for the year 1970, and then examine the relationship between housing price and demand in the US from 1947 to 2007. They conclude that the elasticity of demand is 5.3%, suggesting that a 1% increase in demand for housing leads to a 5.3% increase in the real price of housing.

A number of papers<sup>11</sup> analyze the elasticity of house prices to demographic

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<sup>11</sup> [28], [52], [57], [59], [69], and [71].

variables. The various demographic variables used in these papers are the old age dependency ratio, the proportion of the population of working age, the proportion of the population in their twenties, and the total population. As mentioned above, Mankiw & Weil [52] use an age-specific demand value from cross-sectional regressions. In the papers that use the old age dependency ratio, its elasticity to house prices is negative and ranges from -0.6625 to -1.3167 at the 1% level of significance. These papers access data from the US, Japan, Hungary, and the OECD countries.

Takáts [71] is based on 22 advanced economies. He concludes that a 1% increase in the old age dependency ratio is associated with a drop in house prices of 2/3%. In contrast, Saita et al. [69] examine data from the US and Japan. They find larger elasticities of -0.9% for the US and -1.33% for Japan. On the other hand, Jäger & Schmidt [43] do not find a significant relationship between age structure and house prices in their analysis of panel data for 13 developed economies.

Using the total population as the demographic measure, Takáts [71] finds an elasticity to real house prices of 1%. Using the same demographic measure, Farkas [28] finds this elasticity to range from 3% to 4%.

Kraft & Munk [48] is not directly related to demographic structure. They conclude that expected individual housing investment increases during working life and gradually falls during retirement. Mayordomo et al. [57] use Spanish data to analyze mortgages and housing based on age groupings. They conclude that households under age 35 have the highest level of mortgage; retired households have the lowest.

Černý et al. [17] and Guirguis et al. [36] follow quite different approaches. Černý et al. [17] analyze UK data, taking individual age as the demographic

variable. They examine the owner occupancy rate and the value of owner occupancy. Guirguis et al. [36] does not detect any significant relationship between their demographic variable (ratio of those age 25 - 35 to total population) and real housing prices.

## 2.7 Infrastructure

As noted above in Section 2.1, there were no papers found that provided any connection between infrastructure returns and demographic variables. In fact, we found only five papers that dealt with infrastructure as a class of assets. This Section provides a brief summary of those papers in chronological order.

Idzorek & Armstrong [40] study the role of infrastructure in the strategic asset allocation for portfolios. Using a variety of infrastructure indices, they conclude that infrastructure should be treated as a separate asset class for potential inclusion in strategic asset allocations. Including infrastructure provides only a modest improvement in the risk/reward trade-offs within a portfolio. However, an unconstrained optimization will add a material allocation to infrastructure (up to 6% of the portfolio).

Inderst [42] provides a very detailed primer on infrastructure.

Ben Ammar et al. [7] develop common risk factors for infrastructure, much like Fama & French [24] do for equity and bonds. Their seven risk factors are the broad market, cash-flow volatility, leverage, investment growth, term, default, and regulatory risks. They test their model with data from specific sectors in the equity market: utilities, telecommunications, and transportation. They find that their model does a good job, in particular doing a better job than the Fama & French [24] three-factor model and the Carhart [15] four-

factor model.

Bianchi et al. [8] employ the Carhart [15] four-factor framework to analyze US infrastructure returns. They create historical infrastructure returns by using a variety of indices and the time period 1927 to 2010. They find that the returns exhibit low market beta and a value premium. However, they also find significant variability in the results depending on which of the indices they use.

Finally, Bianchi et al. [9] analyze Australian data on infrastructure from four different indices. They conclude that using a simple historical mean return does a better job of modelling future infrastructure returns than any conventional asset pricing model.

## 2.8 Conclusion

This literature review provides a summary of over 60 academic and other articles that explore the linkage between demographic factors and asset class returns. These articles cover a range of time periods and geographic areas. While the details of their results vary, the general conclusion that is often reached is that aging populations will dampen the future returns on equities, bonds, and housing. The following points try to summarize the effects identified in the literature:

- An increase of 1% in the proportion of the population that is working aged, increases equity returns by somewhere in the range of 1.5% to 5% per year.
- An increase of 1% in the proportion of the population that is over age 65, reduces equity returns by roughly 0.5% per year.



- An increase of 1% in the proportion of the population that is working aged, reduces bond yields by somewhere in the range of 1% to 1.5%.
- An increase of 1% in the old age dependency ratio, reduces house prices by somewhere in the range of 0.5% to 1.5%.

As most government statistical agencies currently produce population projections that expect their populations to age, this literature suggests that some downward adjustment should be made to most assumptions about future expected investment returns.

It would also be interesting to know the relative impact of demographic factors and economic factors on the returns of various asset classes. This is difficult to tease out from the literature, and particularly difficult to interpret. However, the following is a summary of the results in Davis & Li [20].

- In their analysis of the change in the log of real share prices for the aggregate of 7 countries, three variables are statistically significant – the proportion of the population between ages 40 and 64, the real long bond yield, and the dividend yield from the previous year. For every percentage point increase in the proportion of the population aged 40 - 64, the log of real share prices increases by 8%. For every percentage point increase in the real long yield, the log of real share prices increases by 2%. And finally, for every percentage point increase in the previous year's dividend yield, the log of real share prices increases by 9%.
- In their analysis of the change in real bond yields for the aggregate of 7 countries, five variables were significant – the proportion of the population between the ages of 40 and 64, the change in the short bond yield, the spread between long and short bond yields in the prior year, and the first and second differences in the log of CPI. For every percentage point increase in the proportion of the population aged 40 - 64,

the real long bond yield reduces by 1.29%. For every percentage point increase in the short bond yield, the real long bond yield increases by 0.50%. For every percentage point increase in the term spread from the previous year, the real long bond yield reduces by 0.72%. Finally, for every percentage point increase in the first and second differences of the log of CPI, the real long bond yield reduces by 116.4% and 143.3%, respectively. As a way to provide this last relationship more conceptual meaning, if CPI had increased from 1.75% to 2.00% to 2.25% over the last three years (i.e., the first difference of CPI is +0.25% and the second difference is nil), then the real long bond yield would be expected to be lower by 0.14%.

From the perspective of a pension plan risk manager, what would be important to know is the relative effect of population aging on stocks and bonds. That is, do changes in population structure affect stocks more or less than bonds? Unfortunately, this seems to be a gap in the existing literature, and research on this topic would be welcome.

Table 2.1: Stock Market Factors as Dependent Variables

Demographic Definition	Stock Market Factor				
	Stock Price	P/E Ratio	Dividend Yield	Market Structure	Market Structure
Age intervals and their ratios	3LT [3] [20] [39] [50] 4LT [11] 7AG [10] 15AG [11] 81AG/SR [45]	[34] [68] [68] [66]	[49] [53] [12] [63] [29] [30] [32] [4]	[19] [61]	
Working age related factors	WA15 WA25 [46] PPEA [5] [60] OAD [3]	[44]	[65] [65]	[35] [35]	
Other Factors	[56]		[38] [47]		

Table 2.2: Summary of Cases and Dates for Stock Market Research

Research	Year of Publication	Case/ Countries	Data Period
[64]	2001	U.S	1910-2000
[20]	2003	7 OECD-countries	UN database (1998)
[3]	2003	Developed Countries	
[50]	2003		
[63]	2004	US	1926-2003
[35]	2004	U.S	
[44]	2004	U.S, U.K	1950-2000
[34]	2004		
[53]	2006	All G7-countries	1951 - 2002
[19]	2006	72 countries	1960-2002
		, ( 23 OECD-countries, 36 EMEs and 13 transition economies)	
[11]	2006	US- Germany- Italy	
[10]	2006	U.S	
[46]	2006	U.S	1950-2004
[39]	2006	Australia	1965-2002
[61]	2007	25 country (OECD)	1980 - 2001
[30]	2009	U.S	1909-2006
[67]	2009	Germany	
[12]	2010	Italy	1958-2004
[60]	2010	U.S	1900-2007
[5]	2010	U.S	1949-2005
[32]	2010		
[29]	2011	U.S	1909-2006
[56]	2011	US	1910 -2006
[4]	2012	22 developed countries	60 years data
[68]	2012	US, UK, Japan, France and Germany	
[25]	2012	23 Countries	1989-2011
[66]	2013	UK	
[49]	2013	U.S	1937-2011
[38]	2013	G7	1950-2007
[1]	2014	U.S	
[57]	2014	Spain	2002-2005
[47]	2014	U.S	1950-2005
[21]	2015	Norway	1995-2009
[31]	2015	US	1964:2- 2007:4
[65]	2016	US	1950-2010

Table 2.3: Bond Market Factors as Dependent Variables

	Bond Yields			Bond Price/Market Size		Interest Rates
Age intervals	3LT	[12] [20] [31] [53] [63] [68]	[11] [19] [61]	[31] [33]		
and their ratios	4LT					
	7AG	[31]				
	15AG	[4] [66]	[11]			
	81AG/SR					
Working age related factors	WA15			[2] [41]		
	WA25			[54]		
	PPEA			[54]		
	OAD			[2]		
Other Factors		[53]		[16] [56]		

Table 2.4: Summary of Cases and Dates for Bond Market Research

Research	Year of Publication	Case/Countries	Data Period
[20]	2003	7 OECD-countries	UN database (1998)
[63]	2004	US	1926-2003
[54]	2005	US, UK, Japan, and Ireland	
[53]	2006	All G7-countries	1951 - 2002
[19]	2006	72 countries	1960-2002
		( 23 OECD-countries, 36 EMEs and 13 transition economies)	
[11]	2006	US- Germany- Italy	
[61]	2007	25 country (OECD)	1980 - 2001
[30]	2009	U.S	1909-2006
[12]	2010	Italy	19582004
[56]	2011	US	1910 -2006
[4]	2012	22 developed countries	60 years data
[68]	2012	US, UK, Japan, France and Germany	
[66]	2013	UK	
[41]	2014	Japan	Since 1975
[31]	2015	US	1964:2- 2007:4
[2]	2015	21 OECD-economies	1970-2007
[16]	2015	Developed Economies	
		(Northern America, Western Europe, Japan, Australia, New Zealand)	
[33]	2017	US	1970-2013

## Chapter 3

# Specification of a Model Incorporating the Impact of Changes in Demographic Structure on the Long-term Dynamics of House Prices

### 3.1 Background

In order to study the impact of demographics on house prices, we proposed to employ the models developed by Takats [71] and Nishimura & Takats [59]. These models rely on an overlapping generations structure that is amenable to studying the impact of unequal generation sizes on the long-term dynamics of asset prices. However, the Nishimura & Takats [59] model assumes that housing supply is fixed, an important assumption that we relax. We also modify the model in other ways to focus on the features of interest to us. We use a more general utility function, but do away with money supply elements that are not necessary for our analysis with respect to incorporating a flexible

housing supply. The purpose of this chapter is to describe a model of housing where its supply is not fixed, and is potentially affected by demographics. The model is described in Section 3.2. The first order conditions are provided in Section 3.3. Section 3.4 sets out an approach to the empirical testing of the model. A glossary of terms used in this document is contained in Table 3.1 at end of the chapter.

## **3.2 Description of Model**

The reader should bear in mind that economic models are “stylized”, abstracting from all but the items of interest. In this model, there are two generations alive at any one time – the young and the old. The young get “mana from heaven” in the form of an endowment that gets created from their work and the involuntary bequests of the previous old generation. This is their “income”. From that income, they can obtain consumption and housing, nothing else. They consume while they are young, so the only thing that they bring forward to when they are old is their savings in the form of housing. When they are old, they take this housing and sell part of it to finance their consumption. The rest they live in and end up leaving to the next generation of young.



$$U(c_t, h_t) = u(c_t^Y, h_t^Y) + \beta u(c_{t+1}^O, h_{t+1}^O) \quad (3.1)$$

$$c_t^Y \leq y_t - h_t^{Purchased} q_t - h_t^{Converted} g(h_t^{Converted}) q_t \quad (3.2)$$

$$h_t^Y = h_t^{Purchased} + h_t^{Converted} g(h_t^{Converted}) \quad (3.3)$$

$$c_{t+1}^O \leq h_{t+1}^{ForSale} q_{t+1} + h_{t+1}^{Empty} g(h_{t+1}^{Empty}) q_{t+1} \quad (3.4)$$

$$h_{t+1}^O = h_{t+1}^{ForSale} + h_{t+1}^{Empty} g(h_{t+1}^{Empty}) - \frac{c_{t+1}^O}{q_{t+1}} \quad (3.5)$$

$$Y_t = f(n_t^Y) + n_{t-1}^O h_{t-1}^O q_t \quad (3.6)$$

$$n_t^O c_t^O = n_t^Y h_t^{Purchased} q_t \quad (3.7)$$

- Formula 3.1 is a more general utility function than in Nishimura & Takats [59], though without any money in the model. Our initial thinking is to use a Constant Relative Risk Aversion (CRRA) sub-utility function with some parameter to weight the relative utility of consumption ( $c_t$ ) and housing ( $h_t$ ). Potentially, the sub-utility function might look like the following:

$$u(c_t, h_t) = \frac{c_t^{1-\alpha}}{1-\alpha} + \Psi \frac{h_t^{1-\gamma}}{1-\gamma} \quad (3.8)$$

$\Psi$  is the parameter that determines the relative utility of consumption and housing.  $\alpha$  is the risk aversion factor for consumption.  $\gamma$  is the risk aversion factor for housing. If  $\Psi$  is set equal to 1, and the coefficients of relative risk aversion<sup>1</sup> are 1 for both consumption and housing, then the sub-utility function reduces to the form used in Nishimura & Takats [59].

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<sup>1</sup>The coefficient of relative risk aversion is defined as  $-\frac{cu''(c)}{u'(c)}$ . So, when the utility function is  $\ln(c)$ , the coefficient of relative risk aversion is constant at 1, and when the utility function is  $\frac{c^{1-\alpha}}{1-\alpha}$  the coefficient of relative risk aversion is constant at  $\alpha$  as long as  $\alpha \neq 1$ .

- Formula 3.2 is the budget constraint for the young. Consumption,  $c_t^Y$ , is what is left over after receiving exogenous income ( $y_t$ ), consuming housing services purchased from the old and priced in consumption units ( $h_t q_t$ ), and potentially converting unused land into housing ( $h_t^{Converted}$ ). The cost of converting unused land into housing is represented by the function  $g(\cdot)$ <sup>2</sup>. Because of the conversion cost, individuals will always prefer to buy housing from the old generation. Only if the old generation does not choose to sell sufficient housing will the young generation convert unused land into housing. The market for unused land is assumed to be competitive, with sellers of the land taking the market price, which is equivalent to the price of housing. That is, unused land is exchanged at the same price as housing. There is only a cost to convert unused land into housing.
- Formula 3.3 is the total housing available for use by each individual in the young generation.
- Formula 3.4 is the budget constraint for the old. The old can consume from housing held over from the previous period, priced in the current period, and sold to the young ( $h_{t+1}^{ForSale} q_{t+1}$ ) and housing held empty at a cost ( $h_{t+1}^{Empty}$ ), where the cost function is the same as the cost to convert unused land to housing. As for the young, the old will always prefer to sell housing to the young generation because of the costs to hold housing empty. They will only hold housing empty if the young do not choose to buy sufficient housing.
- Formula 3.5 is the total housing available for use by the old. It is what is left over from the housing that the old brought from the first period after selling some to the young, potentially leaving some empty, and

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<sup>2</sup>This function serves to replace the fact that we have not limited the amount of unused land. This function would have positive first and second derivatives so that it becomes prohibitively costly to convert more unused land after some point.

selling some for consumption.

- Formula 3.6 provides some functional form for total income. Total income is a function of what the young<sup>3</sup> produce ( $f(n_t^Y)$ ), plus the housing that the previous old generation bequeathed priced currently in consumption units ( $n_{t-1}^O h_{t-1}^O q_t$ ). In our baseline model, we assume that all bequests are involuntary. They are collected by a central planner and redistributed to the next young generation. Then, the total income of the economy is distributed proportionally to the young generation ( $y_t = \frac{Y_t}{n_t^Y}$ ).
- Formula 3.7 is the market clearing condition. The only housing available for the young generation to purchase is what the old generation sells in order to finance their consumption.

### 3.3 First Order Conditions

First, rewrite the model with some functional forms inserted for  $u(c, h)$  and  $f(n_t^Y)$ <sup>4</sup>. All of this is done for ease of exposition. A more general utility function is used in the following subsection.

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<sup>3</sup> $n_t^Y$  and  $n_t^O$  are the total numbers of the young and old generation at time  $t$ , respectively.

<sup>4</sup>In this simplified form, each young person is assumed to produce a constant amount,  $\bar{y}$ .

### 3.3.1 Log Utility

$$U(c_t, h_t) = \ln(c_t^Y) + \ln(h_t^Y) + \beta \ln(c_{t+1}^O) + \beta \ln(h_{t+1}^O) \quad (3.9)$$

$$c_t^Y \leq y_t - h_t^{\text{Purchased}} q_t - h_t^{\text{Converted}} g(h_t^{\text{Converted}}) q_t \quad (3.10)$$

$$h_t^Y = h_t^{\text{Purchased}} + h_t^{\text{Converted}} g(h_t^{\text{Converted}}) \quad (3.11)$$

$$c_{t+1}^O \leq h_{t+1}^{\text{ForSale}} q_{t+1} + h_{t+1}^{\text{Empty}} g(h_{t+1}^{\text{Empty}}) q_{t+1} \quad (3.12)$$

$$h_{t+1}^O = h_{t+1}^{\text{ForSale}} + h_{t+1}^{\text{Empty}} g(h_{t+1}^{\text{Empty}}) - \frac{c_{t+1}^O}{q_{t+1}} \quad (3.13)$$

$$Y_t = n_t^Y \bar{y} + n_{t-1}^O h_{t-1}^O q_t \quad (3.14)$$

$$n_t^O c_t^O = n_t^Y h_t^{\text{Purchased}} q_t \quad (3.15)$$

There are potentially four control variables: consumption while young and old, and housing while young and old. The first order conditions (FOCs) are the first derivatives of the system of equations with respect to each of these control variables.

$$c_t^Y : \frac{1}{c_t^Y} - \frac{1}{h_t^Y} \frac{1}{q_t} = 0 \quad (3.16)$$

$$h_t^Y : -\frac{1}{c_t^Y} q_t + \frac{1}{h_t^Y} = 0 \quad (3.17)$$

These two FOCs are equivalent and simplify to  $c_t^Y = q_t h_t^Y$ .

$$c_{t+1}^O : \frac{\beta}{c_{t+1}^O} + \frac{\beta}{h_{t+1}^O} \left( -\frac{1}{q_{t+1}} \right) = 0 \quad (3.18)$$

$$h_{t+1}^O : \frac{\beta}{h_{t+1}^O} + \frac{\beta}{c_{t+1}^O} (-q_{t+1}) = 0 \quad (3.19)$$

As before, these two expressions are equivalent. They simplify to  $c_{t+1}^O =$

$h_{t+1}^O q_{t+1}$ .

What these two sets of FOCs suggest is that in both generations, though not necessarily across generations, it is optimal to split consumption equally between consumption and housing. This relationship arises as a result of the sub-utility function used.

### 3.3.2 Generic CRRA Utility

$$U(c_t, h_t) = \frac{(c_t^Y)^{1-\alpha}}{1-\alpha} + \Psi \frac{(h_t^Y)^{1-\gamma}}{1-\gamma} + \beta \frac{(c_{t+1}^O)^{1-\alpha}}{1-\alpha} + \beta \Psi \frac{(h_{t+1}^O)^{1-\gamma}}{1-\gamma} \quad (3.20)$$

$$c_t^Y \leq y_t - h_t^{Purchased} q_t - h_t^{Converted} g(h_t^{Converted}) q_t \quad (3.21)$$

$$h_t^Y = h_t^{Purchased} + h_t^{Converted} g(h_t^{Converted}) \quad (3.22)$$

$$c_{t+1}^O \leq h_{t+1}^{ForSale} q_{t+1} + h_{t+1}^{Empty} g(h_{t+1}^{Empty}) q_{t+1} \quad (3.23)$$

$$h_{t+1}^O = h_{t+1}^{ForSale} + h_{t+1}^{Empty} g(h_{t+1}^{Empty}) - \frac{c_{t+1}^O}{q_{t+1}} \quad (3.24)$$

$$Y_t = n_t^Y \bar{y} + n_{t-1}^O h_{t-1}^O q_t \quad (3.25)$$

$$n_t^O c_t^O = n_t^Y h_t^{Purchased} q_t \quad (3.26)$$

The two FOCs for  $c_t^Y$  and  $h_t^Y$  are equivalent and simplify to  $q_t (h_t^Y)^\gamma = \Psi (c_t^Y)^\alpha$ . Also, the two FOCs for  $c_{t+1}^O$  and  $h_{t+1}^O$  are equivalent and simplify to  $q_{t+1} (h_{t+1}^O)^\gamma = \Psi (c_{t+1}^O)^\alpha$ .

A further step in this research will be to analyze the implications of changing cohort sizes within the model.

## 3.4 Approach to Empirical Testing

### 3.4.1 Existing Literature

The aim and scope of this section are to review the empirical tests that have been applied in studies of the relationship between house prices and demographic changes in order to select a preferred approach to test the model set out in Section 3.2. Specifically, we would like to be able to test the relationship between the relative sizes of generations (i.e.,  $n_t^Y/n_t^O$  or its reciprocal) and the change in the price of housing ( $q_{t+1}/q_t$ ). Although much of the literature is based on the seminal paper, Mankiw & Weil [52], the articles have taken different approaches in order to detect a significant relation.

As an asset pricing approach, Chu [18] mentions that owner-occupied housing is the single most important consumption good as well as the dominant asset in most households' portfolios. The demand for owner-occupied housing is thus a combination of intratemporal consumption choice and intertemporal portfolio choice. His cross-sectional Fama-MacBeth regressions show that the conditional models, conditioning on the non-durable consumption of housing ratio, perform much better than their unconditional counterparts. And, the conditional two-factor model derived in his paper performs almost as well as Fama-French three-factor model. Since housing demand is a combination of consumption demand and asset demand, it contains information about the expected returns on traded assets.

Černý et al. [17] find that the demand for owner-occupied housing is sensitive to demographic change and to reform of a PAYGO pension system.

Guirguis et al. [36] consider the population ratio (the fraction of the population between age 25 and 35) as one of the factors in forecasting the volatility of real house prices. They apply other factors such as real disposable income,

the stock of owner occupied dwellings, the expected nominal capital gains, and the nominal post-tax mortgage interest rate in their research.

Kraft & Munk [48] find that before retirement, the optimal strategy involves borrowing via the money market. From about 10 years before retirement and through most of the retirement phase, the total housing investment exceeds the desired housing consumption. This can be implemented by (i) buying more housing units and renting out part of those, or (ii) purchasing a house whose size corresponds to the (average) desired housing consumption for the remainder of life and investing in the house price-related financial assets (quite heavily so just after retirement).

From the aspect of empirical analysis, some researchers consider a specific country [28], [52], [57], and [69], whereas some others investigate multiple countries and apply panel regressions [51] and [71]. Most of the researchers who used regression or time series considered real GDP per capita in their empirical testing [51], [69], and [71]. However, a few researchers also include the interest rate as another variable, or they consider it in their robustness tests [36] and [55].

Fairchild et al. [22] claim that when the central bank lowers the key interest rate, it could stimulate the demand for housing in all markets and have a positive effect on house prices. Based on a theoretical model, Martin [54] claims that house prices and interest rates must have an inverse relationship. Moreover, he shows that the demographic effect of the baby boom is a likely driver of both interest rates and house prices, and that both of these prices are likely to be influenced for some time to come as the baby boomers slowly retire and eventually die.

Farkas [28] is the only one who considers three-year lagged construction

costs as an instrumental variable. He claims that the rationale for using three-year lagged variables is that the design and construction period together for dwellings usually takes less than three years.

Cannon et al. [14] use a panel data set comprised of 7,234 postal ZIP codes falling in 155 urban Metropolitan Statistical Areas (MSAs) across the United States. Socioeconomic data used in the study include median household income, the civilian unemployment rate, the percentage managerial employment, and percentage of owner-occupied housing.

### 3.4.2 Proposed Approach

We propose to use panel data related to cities, as follows:

- Phase 1: Use real house price for each city as the dependent variable. The old age dependency ratio and total population of each province (or state) will be the demographic factors. Real provincial (or state) GDP per capita will be the economic factor.
- Phase 2: If the data were available, we would follow the methodology of Farkas [28] and apply the three-year lagged construction cost and demographic variables as instrumental variables, and compare to the results of Phase 1. Given the difficulty of obtaining construction costs, we expect just to use lagged demographic variables as instrumental variables.
- Phase 3: Apply other demographic factors instead of the old age dependency ratio and total population.
- Phase 4: Check robustness by adding the interest rate (mortgage rate) to the regression model.

Potential sources for the data are contained in Table 3.2.



Table 3.1: Glossary of Terms

$\alpha$	risk aversion factor for consumption
$\beta$	utility rate of time preference
$\gamma$	risk aversion factor for housing
$\Psi$	relative utility of housing in terms of consumption
$c_t^Y, c_t^O$	consumption at time $t$ for the young ( $Y$ ) and the old ( $O$ ) generation, respectively
$h_t^Y, h_t^O$	consumption of housing at time $t$ for the young and the old generation, respectively
$h_t^{Purchased}$	housing that the young generation has purchased from the old generation at time $t$
$h_t^{Converted}$	housing that the young generation has converted from unused land at time $t$
$h_t^{ForSale}$	housing that the old generation has available either for sale to the young generation or to live in at time $t$
$h_t^{Empty}$	excess housing of the old generation at time $t$ that cannot be sold to the young generation due to lack of demand
$q_t$	price of housing in terms of consumption units at time $t$
$n_t^Y, n_t^O$	number of the young (respectively, old) generation at time $t$
$y_t$	income provided to each member of the young generation at time $t$
$Y_t$	total income of the economy at time $t$
$\bar{y}$	amount of income (or GDP) produced by each young person
$g(\cdot)$	function that represents the cost to convert unused land to housing or to hold housing empty. The argument is the amount of housing.
$u(\cdot, \cdot)$	sub-utility function for a particular cohort at a particular time
$U(\cdot, \cdot)$	lifetime utility for a particular cohort

Table 3.2: Sources of Data for Empirical Analysis

Data Element	Canada	United States
House Prices	Teranet and National Bank from July 1990	Zillow from Aug. 2008
CPI	StatsCan available by Province	various sources, including Bureau of Labor Statistics
Demographic Variables	StatsCan available by Province	census.gov available by state
Mortgage Interest Rates	StatsCan	hsh.com
Construction Costs	Not readily available	Not readily available

## Chapter 4

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