FINAL

VALUATION TECHNIQUE
PAPER #2
THE VALUATION OF INDIVIDUAL RENEWABLE TERM INSURANCE

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Canadian Institute of Actuaries **Institut Canadien des Actuaires**

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To: MEMBERS OF THE CANADIAN INSTITUTE OF ACTUARIES

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Enclosed is a copy of the above captioned paper and its appendices. It has been developed by the Financial Reporting Committee of the Institute and was sent to all members as an exposure draft on December 16, 1985. Copies of appendices A and B were issued in early 1986 and sent to Valuation Actuaries. Written comments have been received and discussion of the contents of the paper was held at our June meeting in Vancouver.

At its meeting of September 26, 1986 Council approved Valuation Technique Paper #2 as a binding standard of practice for the Institute, to be used for valuations beginning at year end 1986. Any questions on application of the techniques illustrated in the paper should be directed to Jim Brierley, Chairman of the Financial Reporting Committee.

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I. INTRODUCTION

This paper describes the consideration involved in the valuation of individual renewable term insurance products.

The paper concentrates on valuation considerations which are unique to this type of product. After discussing some considerations which are common to most types of renewable term plans, the paper addresses the valuation of single scale products (see Section III for definition). Special considerations for re-entry products are then discussed. Examples of the valuation of single scale and re-entry products appear as appendices.

Although the following remarks do not apply on to the valuation of individual renewable term insurance products, the valuation actuary is reminded that:

1. The valuation premium must be the lesser of (i) the gross premium, and (ii) the sum of four component valuation premiums, as described in Recommendation 4.09.

2. The valuation actuary should test the sensitivity of reserves to changes in assumptions, and take particular care in selecting any assumption for which a small change could have a major impact on reserves.

3. Recommendation 3.09 requires that the valuation actuary makes a provision for adverse deviations which are larger (compared to expected experience) when he is less confident of the expected experience. In this paper this applies in particular to the assumptions for Selective Lapse Rates (page 6) and Re-entry Proportions (page 9).

II. SUMMARY

1. Benefits, premiums and expenses should be valued to the end of the benefit period, not to an earlier renewable date (page 4).

2. It is acceptable to treat the excess of heaped commissions at a renewable date over normal renewable commissions in the same fashion as issue expenses (page 5).

3. A method of calculating the net valuation premium when the gross premiums are not level, for the purpose of determining the maximum deferrable issue expenses, is described on page 5.

4. Lapse rates can be expected to show a sudden (temporary) increase when premium rates increase at a renewable date. In general, healthy lives are more likely to lapse their policies at renewal than unhealthy lives, the net effect being a deterioration in mortality for the remaining lives.

   Factors influencing these additional lapses are described on pages 6 to 8.

5. In valuing re-entry products, it is necessary to make an assumption about the percentage of policyholders who will re-qualify for select rates at each renewal date. This percentage is referred to as the “Re-entry Proportion”.

   Ignoring Selective Lapses, the mortality for the group of policyholders as a whole should resemble the normal select/ultimate mortality one would expect from a single scale product. For example, the mortality rate in the sixth policy year for the total group will be $q_{[x]=5}$. The gross premium, however, will be:

   $$RP \times GPS + (1 - RP) \times GPU$$

   where ***RP*** = Re-entry Proportion
   GPS = Select (preferred) renewal rate
   GPU = Ultimate (guaranteed) renewal rate

   By using this technique, the valuation actuary no longer has to make an explicit assumption regarding the mortality of those that do not re-qualify for select rates. Instead, the assumption with respect to the Re-entry Proportion, which is required in any case to determine further revenue, leads to the mortality assumption for the lives who do not re-qualify.

   Factors to be considered when choosing the Re-entry Proportion are described on page 10.

III. DEFINITIONS AND SCOPE

A. Definitions

1. "Renewable" means that premium rates are not level for the entire term of the policy, but change at predetermined dates ("renewal dates").

2. "Single scale" means that the premium rate at a renewal date is based only on attained age, and is the same as the rate for a new policy issued at the same age (assuming new policies are still being issued on that rate scale). The word "aggregate" is sometimes used with the same meaning.

3. "Re-entry" means that the premium rate at one or more renewal dates is dependent upon the underwriting classification of the policyholder at such renewal dates. "Revertible", "Multiple Scale" and "Select and Ultimate" are sometimes used with the same meaning.

B. Scope

This paper considers principles applicable to the valuation of individual renewable term insurance products, both single scale and re-entry, whether directly written or assumed by reinsurance. Annual or yearly renewable direct written term products are included. YRT reinsurance, of either renewable or non-renewable direct written products, is also included.

The paper covers adjustable premium products and non-guaranteed premium products if they could otherwise be classified as Renewable Term. For such products the member is reminded of Recommendation 4.07 which requires him to make an assumption about premiums after the guaranteed period. The methods for choosing an assumption for the gross premiums in such cases is outside the scope of this paper.

The following types of products are not covered by this paper:

- Level premium term insurance
- Deposit term insurance

The valuation of any conversion privileges is not covered in this paper.

IV. VALUATION CONSIDERATIONS

A. General Considerations

1. Valuation Period

Benefits, premiums and expenses (including commissions) should be valued to the end of the benefit period (policy expiry date), not to an earlier renewal date, regardless of the existence of premium rate guarantees (see Recommendation 4.07). This approach is preferable to valuation as "n year level term" (where n is the number of years between renewal dates) with an explicit liability for the renewal provision, because the latter method has the following deficiencies:

(a) If the value of the liabilities after the next renewal date is in excess of the value of the corresponding gross premium, a deficiency reserve may be overlooked.

(b) The determination of the liability for the renewal provision may be arbitrary and/or unjustifiable.

(c) Profits in each policy year after the first are unlikely to emerge in proportion to gross premiums.

(d) It is unlikely that the slope of the premium rates will exactly match the slope of the valuation benefits and expenses. Unless the entire benefit period is studied, the reserve at any time after issue may be understated.

For example, even if at the issue the present value of benefits and expenses equals the present value of premiums (i.e., deficiency reserves are not present and the net valuation premium is being used), if mortality rates increase more rapidly than net premiums then, at renewal dates, the present value of benefits and expenses will exceed the present value of premiums.

In some cases it may be appropriate to use simpler methods but only as an approximation in accordance with Recommendation 1.07, and provided the appropriateness of the methods has been confirmed by testing. For example, if a Y.R.T. product has gross premium rates which are higher in every policy year than valuation benefits and expenses, then it may be appropriate to hold the unearned premium as the reserve if testing shows that the theoretical reserve is always less than the unearned premium, and that no material distortion of income will result.

2. Treatment of issue expenses

The Insurance Companies Act are not clear on how issue expenses are to be handled when premiums are not level. The following approach is believed to be in accord with the principles in the acts and has the advantage of giving consistent treatment to different premium structures: level, increasing or decreasing.

(a) Issue Expenses

The CIA Recommendations set down what can be considered as issue expenses. It is peculiar to renewable term that heaped commissions are often paid at renewable dates. It is acceptable to treat the present value of the excess of the heaped commissions over the usual annual commissions in the same fashion as issue expenses, although the payment is not made until well after issue.

Regardless of whether these excesses are treated in the same fashion as issue expenses, provision must be made for them in the valuation. Only the excesses need be included if the valuation premium is decreased by an allowance for the usual annual commissions; all future commissions should be provided for if the valuation premium is not so decreased. To do otherwise is to ignore significant future liabilities. The problem is exacerbated if the valuation premium is increased to defer these excesses without provision for the offsetting liability.

B. The Maximum Allowable

The Insurance Companies Acts allow issue expenses to be deferred to a maximum of 150% of the net level premium. Although the word "level" contemplates an underlying level gross premium, it is not necessary to develop another method when the gross premium is not level. For the purpose of determining the 150% limit, the net level premium can be calculated, using the valuation assumptions, as the present value of the policy benefits to the end of the benefit period divided by an annuity of 1 per annum during the premium-paying period.

Once the amount of the deferrable issue expenses is determined, the valuation premiums can be calculated in accordance with Recommendation 4.09. The amortization of issue expenses does not result in a constant addition to the valuation premium. Rather the valuation premiums should display a constant ratio to the gross premiums in all policy years after the first. Thus the valuation premiums will not be level to the end of the benefit period.

B. Single scale products

1. Mortality deterioration

Lapse rates can be expected to show a sudden, temporary increase when premium rates rise at a renewal date. In general, healthy lives are more likely to lapse their policies at renewal than unhealthy lives, the net effect being a deterioration in mortality for the remaining group of lives.

The lapses at renewal date can notionally be divided into three groups:

(a) Those reflected in the experience underlying the valuation mortality table.

(b) Additional lapses whose mortality experience would be the same as that of all policyholders just before the renewal date.

(c) Additional lapses whose mortality experience would be the same as newly selected lives. Lapses by this group, i.e., by healthy or select lives, are referred to as "Selective Lapses".

The "Selective Lapse Rate" is the ratio of Selective Lapses to business inforce immediately prior to renewal date.

The deterioration in mortality for the remaining lives ("mortality drift") at each renewal will lead to increased reserves at all durations. The mortality drift is obviously dependent upon the Selective Lapse Rate, with mortality deteriorating more rapidly as the Selective Lapse Rate increases.

The following factors should be considered when determining the Selective Lapse Rate assumption(s).

(a) Percentage increase in rates and/or steepness in the rise (large increases are more likely to result in higher Selective Lapses).

(b) Duration since last premium increase (if a long time, Selective Lapse Rates are likely to be higher than if a short time).

(c) Selective Lapse Rates are likely to decrease with duration for the same attained age (i.e., the Selective Lapse Rate at age 45, for someone who took out a policy at age 25, will probably be less than the rate at age 45 for someone who took out a policy at age 35). Although this might suggest that Selective Lapse Rates will, in general, decrease with duration, item (a) above may offset this.

(d) Policy size (larger policies are likely to experience higher Selective Lapse Rates).

(e) Distribution system used (does it have high replacement activity and/or operate in an upscale market? Both factors will lead to higher Selective Lapse Rates).

(f) The existence of heaped commissions at a renewal date (Selective Lapse Rates are likely to be higher if there are no heaped commissions).

(g) External market conditions at the time of renewal (the actuary should anticipate some market pressures which could result in selective lapses).

(h) Number of healthy lives left. As an extreme example, if there are no healthy lives left, the Selective Lapse Rate will be zero. Similarly, the Selective Lapse Rate cannot be greater than the proportion of lives (just before renewal) who are select.

(i) Conversion rates (high conversion rates are likely to improve the mortality of the remaining lives at later durations).

Selective Lapses may occur at times other than the renewal date (for example, in the case of major replacement activity). If high “normal” lapse assumptions are used, it should be assumed that some of those lapses are Selective Lapses.

Appendix C discusses mortality deterioration in more detail and contains a formula for determining the mortality rate for the remaining lives, as well as an APL program.

The approach that follows is simpler and produces mortality rates higher than those developed using the formula in Appendix C. It may be used as an approximation which conforms with Recommendation 1.07.

x is the issue age; s is years since issue.

q_x is the valuation mortality table appropriate for the original group of insured lives.

q'_{x+s} is the mortality rate of all insured lives just prior to the renewal date. At the first renewal date, $q'_{x+s} = q_{[x]+s}$

q''_{x+s} is the mortality rate of the remaining lives after the renewal date.

SL is the additional Selective Lapse Rate by healthy (select) lives at a renewal date (as defined in paragraph (c) at the top of page 6).

AL is the additional lapse rate at renewal by policyholders whose mortality experience is the same as that of all policyholders just before renewal (as defined in paragraph (b) at the top of page 6).

Then: $q'_{x+s} = SL\ q_{[x+s]} + AL\ q'_{x+s} + (1 - SL - AL)\ q''_{x+s}$

$$q''_{x+s} = \frac{(1 - AL)\ q'_{x+s} - SL\ q_{[x+s]}}{1 - SL - AL} \qquad (A)$$

This formula provides the mortality rate of the remaining lives immediately after a renewal date. It is now necessary to consider the future mortality experience of this group.

The mortality rate of the total group of lives immediately before renewal would eventually (at the end of the select period) become the ultimate mortality rate. The mortality rate of the Selective Lapses, although initially select, will eventually become the ultimate mortality rate too. Consequently, the mortality rate of the remaining lives must eventually become the ultimate mortality also.

Define K such that $\frac{q''_{x+s}}{q''_{[x]+s}} = \frac{100 + K}{100}$

Assume the select period is 15 years.

Then

$$q''_{x+s} = \frac{(100 + K)\ q_{[x]+s}}{(\ 100\ )}$$

$$q''_{x+s+1} = \frac{(100 + 14K/15)\ q_{[x]+s+1}}{(\ 100\ )}$$

$$q''_{x+s+2} = \frac{(100 + 13K/15)\ q_{[x]+s+2}}{(\ 100\ )}$$

$$q''_{x+s+5} = \frac{(100 + 10K/15)\ q_{[x]+s+5}}{(\ 100\ )}$$

Assuming there is a five year period between renewal, q''_{x+s+5} would be set equal to the q' factor in formula (A) on page 7 at the next renewal date, and a new value of q''_{x+s+5} (mortality of the remaining lives after the next renewal date) would be determined.

An example of the valuation of a single scale product appears in Appendix A.

Note: If the Valuation Actuary considers the mortality deterioration to run off over more than 15 years, he can construct an underlying table with a longer select period. It would not be proper to use a select period of less than 15 years, particularly for the higher ages.

C. Re-entry products

1. Selective lapsation at renewal

Total lapse for a re-entry product (both selective and non-selective) at renewal dates may be higher or lower than for a single scale product, depending upon company practices and market conditions.

The Selective Lapse Rates for a re-entry product will be affected by the following, in addition to the factors discussed above for single scale products:

(a) The company's underwriting rules and practices at renewal. If the policyholder is required to supply full evidence in satisfying the company's underwriting requirements, he may decide to look for better rates elsewhere. Merely the need to do something may cause the agent and/or the policyholder to review the product for current value and appropriateness.

(b) The difference in agent's compensation between that paid on a select rate, on an ultimate rate, and on a new policy.

(c) The policyholder will become aware of the lack of guarantees in the premium rates and may lapse the product and buy a guaranteed rate product while he is still healthy.

(d) Unhealthy lives will become aware that they are no longer standard lives, and thus will be less likely to lapse their policies.

In general, the proportion of total lapses which are Selective Lapses will be higher for a re-entry product than for a single scale product.

2. The "Re-entry Proportion"

In valuing re-entry products it is necessary to make an assumption either about the mortality of those who do not re-qualify for select rates, or about the percentage of policyholders who will re-qualify for select rates at each renewal date - the "Re-entry Proportion".

It is easier to choose the Re-entry Proportion than to make an assumption about the mortality rate for those who do not re-qualify, because the Re-entry Proportion is related to current and expected future underwriting practice. Many companies will have statistically significant data which they can and should monitor. It should also be noted that the Re-entry Proportion will be required in any event to calculate future revenue.

It is also possible to value the select and ultimate groups separately. However, there is little to recommend this approach since it is difficult even under ideal conditions, and nearly impossible if conditions are less than ideal. It is necessary to make separate mortality assumptions for the select and ultimate groups. Furthermore it will be necessary to study separately the mortality experience of each group of lives that fails to re-qualify for at least 15 years. If the underwriting at re-qualification is the same as at issue, the mortality assumption for re-entry group can be normal select mortality, and the mortality assumption for the ultimate group can be determined by treating the re-entry as a selective lapse. If, as may well be the case, the underwriting is less strict at re-entry, then the mortality for both the select and ultimate groups will be higher than it would otherwise be, but there is no good way of determining an appropriate mortality assumption for either group.

It should be noted that the choice of the Re-entry Proportion leads to an implicit assumption about the mortality of those who do not re-qualify, and vice versa. If the mortality rate for those who do not re-qualify is chosen, the corresponding Re-entry Proportion can and should

be calculated and rationalized. That Re-entry Proportion should also be used in blending premium rates. If the Re-entry Proportion is chosen, the corresponding mortality rates for those who do not re-qualify should be calculated as an overall test of reasonableness. These mortality rates may be very high (e.g. in excess of 1,000% of standard).

If the Re-entry Proportion approach is used, the valuation method can assume that the mortality for the group of policyholders as a whole resembles the normal select/ultimate mortality one would expect from a single scale product given the same Selective Lapses. For example, the mortality rate in the sixth policy year for the total group will be $q_{[x]+5}$. The gross premium, however, will be:

$$RP \times GPS + (1 - RP) \times GPU$$

where RP = Re-entry Proportion
GPS = Select (preferred) renewal rate
GPU = Ultimate (guaranteed) renewal rate

Commissions in the sixth policy year would be determined in a similar fashion.

The following factors should be considered when choosing the Re-entry Proportion.

(a) Attained age. This is often one of the most important factors. The Re-entry Proportion is likely to decrease as attained age increases, due to normal deterioration in health.

(b) The company's renewal underwriting rules, practices and experience. The valuation actuary should consider whether actual practice will be as stringent as intended practice.

(c) The ratio of ultimate rates to select rates to select rates. (If the ratio is close to one, some policyholders may not bother to submit new medical evidence.)

(d) The length of time between re-qualification dates. (The shorter the duration, the higher the Re-entry Proportion.)

(e) The Selective Lapse Rate. (The higher the Selective Lapse Rate, the lower the Re-entry Proportion.) The assumptions for Selective Lapse Rates and Re-entry Proportions should be consistent.

3. Other Considerations

(a) Provision should be made for any renewal underwriting expenses under a re-entry product.

(b) Some re-entry products allow the opportunity to obtain select rates at any time between renewal dates. If this situation exists, it should be assumed that the option will be elected. For example, an annual renewable term product, providing contractual re-entry every five years, in effect allows re-entry every year and therefore should be valued as such.

An example of the valuation of a re-entry product appears in Appendix B.

D. Other Considerations

1. Portfolio revaluation

The techniques described earlier for the handling of Selective Lapses may also be used

when portfolios of aggregate business are (or have been) heavily replaced by non-smoker policies. The excess of actual lapses over normal lapses may be used as the SL factor in the formula on page 7.

2. Dishonesty

   Concerns have been expressed about the level of honesty among those policyholders who declare themselves to be non-smokers. One way of allowing for dishonesty is to blend a percentage of the smoker mortality rate into the theoretical non-smoker rate of produce an expected non-smoker experience rate.

3. Deficiency reserves and interest rate assumption

   If deficiency reserves need to be set up, this creates an opportunity to match asset and liability durations, thus decreasing the risks associated with reinvestment and disinvestment. The existence of matching, combined with the fact that long term rates are usually higher than short term rates, may permit the use of a higher interest rate assumption.

4. Lapse rates for Y.R.T. reinsurance

   When valuing Y.R.T. reinsurance, the lapse rates assumptions depend upon the features of the direct product, not the reinsurance treaty.

APPENDIX A

HYPOTHETICAL EXAMPLE OF THE VALUATION OF A SINGLE SCALE PRODUCT

INTRODUCTION

The purpose of this example is to demonstrate how the principles of this paper might be followed in the valuation of a single scale five year renewable and convertible term product. The example is hypothetical, although there are plans with gross premiums close to those used.

In this example, the first step is to make the preliminary valuation assumptions. Sensitivity tests are then performed, in accordance with Recommendations 3.03 and 3.09, for most of the assumptions. (Not all the sensitivity tests or the results are included in the text.) Before the final valuation is done some of the preliminary assumptions are refined because of insights gained from the sensitivity tests.

The valuation assumptions referred to in this example are illustrative only. Their description here should not be construed as a general endorsement.

DESCRIPTION OF PLAN

The plan to be valued is called “Single Scale 5”. The plan provides level death benefits to age 75. The premiums are guaranteed throughout and are level for each 5-year period at an amount depending on the attained age of the life insured at the renewal date. The plan is also convertible, prior to age 65, into any permanent plan.

Single Scale 5 is sold through brokers who operate predominantly in an upscale market of relatively sophisticated buyers. The pricing of this product was based on market rates.

Single Scale 5 was introduced for sale on January 1 of the current year.

GROSS PREMIUMS

The following table gives a subset of the gross premium rates for Single Scale 5. All the rates in the table below are for male non-smokers. The rates in the rightmost column are the ones used in the valuation of the life insurance benefit. The middle column includes the premium for the conversion privilege, which was assessed to be 4% of the total premium for ages younger than 65. The premium for the conversion privilege is used in the valuation of that privilege. This latter valuation is outside the scope of this paper.

AGE	GROSS PREMIUM RATE PER M	PREMIUM RATE EXCLUDING CONVERSION
25	$1.05	$1.01
30	1.15	1.10
35	1.25	1.20
40	1.75	1.68
45	2.65	2.54
50	4.30	4.13
55	5.95	5.71
60	9.60	9.22
65	16.25	16.25
70	27.00	27.00

There is a policy fee of $50 charged each year.

PRELIMINARY VALUATION ASSUMPTIONS

1. Average Size

   To date the average size in the $250,000-$500,000 size band has been $275,000. For valuation purposes, the average policy size is assumed to be $250,000.

2. Mortality

   The determined of an appropriate underlying non-smoker mortality table is outside the scope of this paper. It was assumed that non-smoker mortality can be represented as a constant percentage (70% was used) of the CA 1969-75 table, although using a constant percentage is too simplistic for actual valuations.

   This basic assumption needs to be modified to account for deterioration in mortality following renewal dates. The mortality of the persisting lives is calculated using the method described in Appendix C.

3. Interest

   The investment strategy for this plan is to use the company's average money portfolio, which is made up of relatively short term investments (less than 5 years). This portfolio is now earning 11% net of investment expenses, and the new money rates are also around 11%. The valuation interest rate starts at 10%, and decreases by 0.5% per year to an ultimate interest rate of 5%.

4. Lapses

   The valuation lapse rates are determined as the sum of base lapse rates applicable to all policy years and additional lapse rates applicable only at renewal dates.

   As an aid to setting the preliminary lapse assumption, a lapse study was done on the company's term insurance policies in the last four policy years.

   Underlying Lapse Rates

   Ideally the Underlying Lapse Rates for the first three or four policy years would come from the lapse study more or less directly. However, during the period covered by the study, lapses were rather higher than can be expected to continue because of the move to rates separated between smokers and non-smokers. The experience lapse rates are approximately adjusted for this effect in determining the preliminary valuation assumption.

   The rate for the fifth and later policy years is chosen as a compromise between the lapse rates that could reasonably be inferred to have underlain the CIA 69-75 table and the lapse rate that could be anticipated for a term insurance plan which was not subject to rate increases at renewals. It will not be known until after the sensitivity testing is done whether a sizeable margin for adverse deviation is required in this assumption, or whether such a margin is provided for by higher or lower lapse rates.

DURATION	UNDERLYING LAPSE RATE
1	15%
2	12
3	9
4	7
5+	5

Additional Lapse Rates at Renewal Dates

The additional lapse rates reflect the increased tendency to lapse at renewal dates when premiums are increased. The lapse rates reflect the opportunities to shop for better rates (primarily at younger and middles ages) and the effect of large increases in premiums and/or reduced need for insurance at the older ages. They are assumed to be zero at other durations.

Because of the lack of experience data, considerable sensitivity testing will be done to ensure that margins are appropriate.

ATTAINED AGE	ADDITIONAL LAPSE RATE	REMARKS
30	10.0%	some shopping for better rate
35	12.5	
40	15.0	
45	17.5	more incentive to shop because of
50	20.0	greater increase in premium
55	22.5	
60	25.0	
65	27.5	reduced need for insurance
70	30.0	reduced ability to pay

The additional lapses in the above table must now be separated into Selective lapses and Average lapses (as defined in Appendix C).

Selective Lapse Rates

The Selective Lapse Rates are calculated by multiplying the additional lapse rates in the above table by the Selective Proportions which are given in the following table. Of course, the Average Lapse Rate is determined by subtraction.

DURATION	SELECTIVE PROPORTIONS	REMARKS
5	90%	almost all still healthy
10	80	
15	70	
20	60	
25	50	
30	50	many healthy lives already gone
35	50	

For example, the application of the above tables yields the following lapse rates for issue age 25.

DURATION OF POLICY	TOTAL LAPSE RATE	BASE LAPSE RATE	ADDITIONAL LAPSE RATE AT RENEWAL	SELECTIVE LAPSE RATE	AVERAGE LAPSE RATE
1	15.0 %	15.0%	0.0%	0.0%	0.0%
2	12.0	12.0	0.0	0.0	0.0
3	9.0	9.0	0.0	0.0	0.0
4	7.0	7.0	0.0	0.0	0.0
5	15.0	5.0	10.0	9.0	1.0
6-9	5.0	5.0	0.0	0.0	0.0
10	17.5	5.0	12.5	10.0	2.5
11-14	5.0	5.0	0.0	0.0	0.0
15	20.0	5.0	15.0	10.5	4.5

5. Premium-Related Expenses

Commissions are 60% in the first year, 25% in quinquennial renewal years, and 5% in other renewal years, and are applied to policy fees as well. Production bonuses in the first year average 30% of premium. The company's marketing support expenses are also allocated to first year premium and amount to 20% of premium. Indirect expenses allocated to premiums are 8% in all years. Premium tax is 2%.

Since the valuation can include as deferrable issue expenses only those expenses which vary directly with the amount of new business, not all the marketing support expenses (we have assumed 50%) can be included (Recommendation 3.12). Part of the first year commission, equal to the non-heaped renewal commission, is considered as an administrative expense.

The valuation assumptions for premium-related expenses are issue expenses of 95% [60+30+(.5x20)-5] in the first year, and administrative expenses of 15% (5+8+2) in each year. Issue expenses include the production bonuses and half of the marketing support expenses, but are reduced by the non-heaped renewal commission of 5%. The administrative expenses include non-heaped renewal commissions, overhead expenses and premium taxes.

At each quinquennial renewal, a further issue expense of 20% of the premium is assumed. (25% of commissions less non-heaped renewal commission of 5%.)

6. Other Expenses

Expense studies indicate that underwriting and policy issue costs average $150 per policy in the size range assumed, and administrative costs are $15 in all years.

The valuation assumptions for non-premium related expenses are $150 per policy as issue expense and an annual administrative expense of $20 ($5 for margin) inflated at 3% per year.

Because of the large average size policy this assumption will not be particularly significant.

SENSITIVITY TESTING

The following table shows mean reserves calculated using the above assumptions on the 1978 Canadian Method. (The formula is given on page A-12.) The weighted average is calculated by weighting the reserves for the various issue ages.

BASE CASE (1978 Canadian Method)

ISSUE		DURATION						
AGE	WEIGHTS	0.5	1.5	4.5	9.5	14.5	19.5	24.5
25	20%	-.50	-.23	1.08	3.75	7.20	12.10	19.36
35	30	1.76	2.74	5.71	11.82	20.08	31.65	42.80
45	30	5.24	7.51	13.82	29.64	46.70	59.29	61.24
55	20	7.75	11.59	21.48	43.58	68.07	26.96	-
AVG.		3.35	5.53	10.37	21.90	35.09	35.09	43.85

Although the final valuation will be done using the 1978 Canadian Method, this method is not always the best to use for sensitivity testing. Some of the sensitivity can be hidden through the calculation of net valuation premiums and by the presence or absence of deficiency reserves. Instead the testing was done using the Policy Premium Method. (The formula is given on page A-11). The Policy Premium Method calculates a reserve which is simply the present value of all future benefits and expenses less the present value of all future premiums. It is not necessary to calculate a net premium, and there is no deferral of acquisition expenses, per se. Any profit or loss, relative to the valuation assumptions, is "realized" at issue. Terminal reserves are shown rather than mean reserves for convenience.

The Policy Premium terminal reserves shown in the following table represent the base case for the scenario testing. The assumptions described above were used in the calculation.

BASE CASE (Policy Premium Method)

ISSUE	DURATION							
AGE	0	1	2	5	10	15	20	25
25	0.57	-0.92	-0.56	0.99	4.18	8.47	14.65	24.07
35	2.77	1.83	2.88	6.78	14.43	25.13	40.40	55.35
45	6.59	5.68	8.03	17.52	38.53	61.79	76.49	73.72
55	10.07	7.81	11.79	27.89	57.52	89.09	-	-
AVG.	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42

1. Mortality

   The effect of an increase or decrease of five percentage points in the mortality ratio (70%) for the underlying mortality table is as follows, for the weighted average:

	DURATION							
	0	1	2	5	10	15	20	25
+5%	6.18	5.13	7.27	15.96	32.93	52.11	54.43	62.57
Base	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42
-5%	3.68	2.12	3.75	10.13	23.48	39.00	40.47	46.18

   Recommendation 3.09 would suggest that a higher margin be required if the portfolio were relatively less mature since the sensitivity to the underlying mortality assumptions is greater at the early durations.

2. Interest

The impact of changing the interest assumption at all durations is as follows:

	DURATION							
	0	1	2	5	10	15	20	25
-0.5%	5.30	4.08	6.05	13.93	29.46	46.92	48.97	55.78
Base	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42
+0.5%	4.61	3.22	5.03	12.27	27.08	44.33	46.09	53.12

Sensitivity to a change in the interest rate is greater (as a percentage of the reserve) at the earlier durations and younger issue ages. Recommendation 3.09 would therefore require a larger margin for a relatively young or immature portfolio.

3. Underlying Lapse Rates

The following table shows the effect of varying the ultimate Underlying Lapse Rate, which applies from duration 5 on. Lapse rates for the first four policy years and the additional lapse rates at renewal are not changed.

	DURATION							
	0	1	2	5	10	15	20	25
-1%	5.57	4.45	6.54	14.93	30.77	48.20	50.42	56.93
Base	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42
+1%	4.40	2.94	4.66	11.48	25.99	43.24	44.85	52.11

Changing the ultimate Underlying Lapse Rate simply changes the rate at which future cash flows are discounted, provided that there is no change made in the rate at which mortality deteriorates. Hence the comments made about sensitivity to interest rate changes apply equally to sensitivity to changes in the Underlying Lapse Rates.

4. The Selective Proportion

The following table shows the effect of varying the selective proportion. In the base case the selective proportion varies from a high of 90% grading down to 50%. For the reserves labeled “high”, the selective proportion grades from 95% to 75%. The “low” case has the selective proportion grading from 75% to 35%. The table also shows the reserves which would result if no mortality deterioration were provided for. This is accomplished by setting the selective proportion at zero; the line is labeled “zero”. The zero case is not a sensitivity test per se, but is included for interest sake.

	DURATION							
	0	1	2	5	10	15	20	25
High	5.30	4.10	6.11	14.26	30.99	51.13	55.43	66.53
Base	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42
Low	4.53	3.10	4.86	11.69	25.23	40.16	41.84	46.92
Zero	3.05	1.19	2.48	6.73	14.88	22.95	26.85	29.52

By increasing the selective proportion the reserves were increased by approximately 7%-10% for all durations and also most ages. The liberalization of the selective proportion caused reserves to be decreased by approximately 10%. Again this ratio was fairly constant by both age and duration.

Relative to some of the other assumptions, the reserves are not very sensitive to the level of the selective proportion. However, du to the lack of data concerning this assumption, Recommendation 3.09 would call for a relatively higher margin. On this basis it was decided that more margin was required in the valuation, and the valuation assumption should be 95% grading down to 60%.

The zero case demonstrates that reserves can be substantially understated if there is no provision for mortality deterioration. Obviously it would not be appropriate to ignore the deterioration of mortality at renewals.

5. Additional Lapse Rates

The following tables show the effect of varying the additional lapse rates, which apply at renewals only. Increases and decreases of 5% are shown. All issue ages are shown as well as the weighted average.

ADDITIONAL LAPSE RATES DECREASED BY 5%

ISSUE	DURATION							
AGE	0	1	2	5	10	15	20	25
25	0.74	-0.71	-0.30	1.38	4.74	9.11	15.30	24.40
35	3.11	2.28	3.44	7.37	14.91	25.02	39.13	52.15
45	7.02	6.23	8.72	17.74	37.03	56.76	67.92	62.98
55	10.04	7.78	11.75	25.95	50.62	73.97	-	-
AVG.	5.20	3.97	5.94	13.00	26.65	41.15	43.97	49.27

BASE CASE

ISSUE	DURATION							
AGE	0	1	2	5	10	15	20	25
25	0.57	-0.92	-0.56	0.99	4.18	8.47	14.65	24.07
35	2.77	1.83	2.88	6.78	14.43	25.13	40.40	55.35
45	6.59	5.68	8.03	17.52	38.53	61.79	76.49	73.72
55	10.07	7.81	11.79	27.89	57.52	89.09	-	-
AVG.	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42

ADDITIONAL LAPSE RATES INCREASED BY 5%

ISSUE	DURATION							
AGE	0	1	2	5	10	15	20	25
25	0.45	-1.07	-0.76	0.69	3.73	7.96	14.15	23.90
35	2.48	1.45	2.41	6.28	14.09	25.47	42.02	59.16
45	6.19	5.15	7.38	17.35	40.31	67.82	86.90	87.23
55	10.04	7.78	11.76	29.95	65.57	108.28	-	-
AVG.	4.70	3.32	5.14	13.22	30.18	51.24	51.88	60.87

Unlike the other assumptions tested, a constant change in the additional lapse rates does not result in a uniform decrease or increase in the reserves. The following comments apply to increasing the additional lapse rate; generally the opposite is true if the lapse rate is decreased.

There are two effects resulting from the higher additional lapse rates. They increase the discounting of future cash flows so that reserves decrease. They also, because the selective

proportion is not changed, result in more mortality deterioration which increase the reserves. The effect of the mortality deterioration does not outweigh the effect of the discounting until later durations. As can be seen in the above tables, the effect various by age. In practice considerably more testing of this assumption would be warranted. The mix of business, both by age and duration, is a very significant factor in assessing the overall importance of the assumption. In this company the assumption turns out to be not too significant.

In practice once a renewal point has been passed, the actual lapse rate should be used to determine the degree of deterioration to date, and only future lapse rates should be adjusted during sensitivity testing.

6. Valuation to the End of the Benefit Period

This is not a matter of sensitivity testing. It is shown for interest only to portray the effect of valuing only to the next renewal date instead of to the end of the benefit period. No specific assumption is made about the reserve for renewability since practices vary widely.

	DURATION							
	0	1	2	5	10	15	20	25
Base	4.94	3.63	5.52	13.06	28.23	45.59	47.50	54.42
5 year	1.02	-1.45	-0.81	0.00	0.00	0.00	0.00	0.00

Not only is it inappropriate to value only to the next renewal date, for reasons given earlier in the paper, but to do so is likely to result in a very serious understatement of reserves.

THE FINAL VALUATION

As a result of the sensitivity testing it was decided to change one valuation assumption; namely, the proportion of the additional lapses at renewal which are to be considered selective, as shown in the following table.

DURATION	SELECTIVE PROPORTION	REMARKS
5	95.00%	almost all still healthy
10	86.25	
15	77.50	
20	68.75	
25	60.00	
30	60.00	many healthy lives already gone
35	60.00	

The following mean reserves were then produced by the 1978 Canadian Method.

ISSUE AGE	WEIGHTS	0.5	1.5	4.5	9.5	14.5	19.5	24.5
25	20%	-.45	-.17	1.18	3.93	7.54	12.72	20.48
35	30	1.87	2.89	5.96	12.35	21.09	33.45	45.83
45	30	5.51	7.85	14.42	30.95	49.23	63.42	66.56
55	20	8.16	12.13	22.42	45.71	71.93	27.92	-
AVG.		3.75	5.61	10.83	22.92	36.99	37.19	47.27

VALUATION METHODS

Two valuation methods were used: the Policy Premium Method and the 1978 Canadian Method. The formulas for the two methods are given in this section.

In both cases the valuation is done to the end of the benefit period rather than simply to the next renewal date. Also in both cases reinsurance is ignored since the adjustment to reserves for reinsurance ceded is outside the scope of this paper. In practice it would not be proper to ignore reinsurance.

THE POLICY PREMIUM METHOD

$$ {}_tV = \sum_{s=t}^{w-1} v^{s-t}\, {}_{s-t}p_x^{T} + t \left[(1000 + \tfrac{1}{2} G_s)\, q_{x+s}^{d} \div \sqrt{1+i_s} + E_s^{k} - (1-E_s^{g})\, G_s \right] $$

where G_s is the gross premium per thousand for year s, including policy fee,

E_s^{k} is the volume-related expense factor for year s,

($E_o^{k} = (150+20)/250$, $E_s^{k} = (20/250) \times 1.03^{s}$ for $s > 0$ in that example),

E_s^{g} is the premium-related expense factor for year s,

w is the duration at expiry, and

v^{s-t} is used as an abbreviation for $\prod_{r=t}^{s-1} \frac{1}{1+i_r}$ (the valuation interest rate is not level).

Note that the premium frequency is annual and withdrawals are assumed to occur at year end.

THE 1978 CANADIAN METHOD

The net level premium is :

$$ N = \left[\sum_{t=0}^{w-1} v^{t+1/2}\, {}_tp_x^{T}\ (1000 + \tfrac{1}{2} G_t)\, q_{x+t}^{d} \right] \div \ddot{a}^{T}_{x:\overline{75-x}|} $$

The amount of deferrable acquisition expense is

$$ DAE = \min \left\{ 1.5N\ ,\ \sum_{t=0}^{w-1} v^{t}\, {}_tp_x^{T}\ \ E_t^{a} \right\} $$

Than the valuation premium in any year is :

$$P_o = min\left\{G_o\ ,\ N_o\right\}$$

$$P_o = min\left\{G_t\ ,\ N_t + (DAE\ \text{x}\ G_t\ /\ (PVG - G_o))\right\}\ \text{, for t > o ,}$$

where $PVG = \sum_{t=0}^{w-1} v^{t}\ {}_{t}p_x^{T}\ G_t$, the present value at issue of all gross premiuns, and

$$N_t = (G_t\ /PVG)\ \text{x}\ \sum_{t=0}^{w-1} v^{s}\ {}_{s}p_x^{T}\left[(1000 + \tfrac{1}{2}G_s)\ q_{x+s}^{d} \div \sqrt{1+i_s} + E_s^{m}\right]$$

The Actuarial Reserve is:

$${}_{t}V = \sum_{s=t}^{w-1} v^{s-t}\ {}_{s-t}p_{x+t}^{T}\left[(1000 + \tfrac{1}{2}G_s)\ q_{x+s}^{d} \div \sqrt{1+i_s} + E_s^{m} + E_s^{a} - P_s\right]$$

The mean reserve is approximated by:

$${}_{t+1/2}V = \frac{\left[{}_{t+1}V\ p_{x+t}^{T} \div \sqrt{1+i_t} + \tfrac{1}{2}(1000 + \tfrac{1}{2}G_t)\ q_{x+t}^{d} \div \sqrt[4]{1+i_t}\ \right]}{(1 - \tfrac{1}{2}q_{x+t}^{d})}.$$

All symbols are as defined for the Policy Premium Method except:

E_t^{a} is the acquisition expense in year ***t***, per thousand

(in this example, $E_o^{a} = 150 / 250\ + .95\ G_o$,

$E_t^{a} = .2\ G_t$ at renewals; and otherwise zero)

E is the maintenance expense per thousand in year ***t***,

(in this example, $E_t^{m} = 1.03^{t}$ x $(20 / 250) + .15\ G_t$)

APPENDIX B

HYPOTHETICAL EXAMPLE OF THE VALUATION OF A RE-ENTRY PRODUCT

INTRODUCTION

The purpose of this example is to demonstrate how the principles of this paper might be followed in the valuation of a five year re-entry term product. The example is hypothetical. The premium rates were selected to be consistent with those of the single scale plan described in Appendix A.

In order to concentrate on the special features of this plan, the pricing and valuation assumptions are generally the same as for the single scale plan studied in Appendix A. Only those assumptions which are different from the single scale plan, namely the Selective Proportion and the Re-entry Proportion, are studied in this Appendix. In practice a number of other assumptions would be different and the broader approach described in Appendix A for the single scale plan should also be followed for the re-entry plan.

As in Appendix A, the first step is to choose preliminary valuation assumptions. This is followed by sensitivity testing, leading to the adoption of a final valuation basis.

The valuation assumptions referred to in this example are illustrative only. Their description here should not be construed as a general endorsement.

DESCRIPTION OF PLAN

The plan to be valued is called “Re-entry 5”. The plan provides a level death benefit to age 75. The premiums are guaranteed at issue and are level for 5 years at an amount depending upon the attained age of the insured and whether or not the insured has qualified for select (preferred) rates at that quinquennial renewal date. The plan is also convertible into any permanent plan before age 65 of the insured.

Re-entry 5 is sold through brokers who operate predominately in an upscale market of relatively sophisticated buyers.

Re-entry 5 was introduced for sale on January 1 of the current year.

GROSS PREMIUMS

The following tables give a subset of the gross premium rates for Re-entry 5. All the rates in the table below are for male non-smokers. The rates used for valuation and shown below are the rates exclusive of the premium for the conversion privilege (4% of the total premium for ages younger than 65). The valuation of the conversion privilege is outside the scope of this paper.

AGE	SELECT	GUARANTEED
20	0.85	1.52
25	0.85	1.52
30	0.94	1.66
35	1.02	1.80
40	1.43	2.52
45	2.16	3.82
50	3.51	6.19
55	4.86	8.57
60	7.83	13.82
65	13.81	24.38
70	22.95	40.50

There is a policy fee of $50 charged each year.

PRELIMINARY VALUATION ASSUMPTIONS

As discussed earlier, all valuation assumptions are the same as for the single scale plan except the following:

1. Selective Proportion

   The following assumptions regarding the proportion of additional lapses which are selective was made:

DURATION	SELECTIVE PROPORTION
5	95%
10	89
15	82
20	76
25	70
30	70
35	70

   This scale of Selective Proportions is slightly higher than that finally adopted in the valuation of the single scale plan.

   Total lapses for a re-entry plan may be higher or lower than those for a single scale plan depending on plan design and the particular circumstances of the company. For the sake of simplicity the same total lapse rate has been assumed for both. However, the actuary should consider the circumstances of his/her company carefully when setting this assumption.

2. Re-entry Proportion

   In order to determine the proportion of policyholders who will be paying select rates, we need an assumption as to the proportion of policyholders who, having qualified for select rates at the last quinquennial renewal date, will also qualify for select rates at the next quinquennial renewal date.

   For valuation purposes, the following preliminary assumptions were made:

ATTAINED AGE	PROPORTION OF SELECT LIVES AT LAST RENEWAL DATE WHO RE-QUALIFY AT NEXT RENEWAL DATE
0-25	.99
30	.98
35	.97
40	.96
45	.94
50	.92
55	.90
60	.875
65	.85

As discussed in the main part of the paper, the actual proportions will depend upon a number of factors, some of them related to the practices of the company. The ratios above are higher than the proportions of new applicants for insurance who qualify for standard rates, but clearly sensitivity testing is required.

Note that, for an insured aged 25 at issue, the Re-entry proportion as defined on page 9 of the main part of the paper (i.e. the percentage of policyholders who will continue to pay select rates at successive renewal dates) is:

START OF POLICY YEAR	RE-ENTRY PROPORTION
6	.98
11	.98 x .97
16	.98 x .97 x .96
21	.98 x .97 x .96 x .94

Because of the lack of experience data, sensitivity testing will be done to ensure that margins are appropriate.

3. Premium-Related Expenses

Commissions are 60% first year, 24% in quinquennial renewal years if the policyholder qualifies for select rates, and 5% in other renewal years. Production bonuses in the first year average 30% of premium. The company's marketing support expenses are also allocated to first year premium and amount to 20% of premium. Overhead expenses amount to 8% of premium in all years. Premium tax is 2%.

Since the valuation can include as issue expenses only those expenses which vary directly with the amount of business, not all the marketing support expenses can be included (Recommendation 3.12). Part of the first year commission, equal to the non-heaped renewal commission, is considered as an administrative expense. The valuation assumptions are issue expenses of 95% [60+30+(.5x20) - 5] in the first year, and administrative expenses of 15% (5+8+2) in each year. Issue expenses include the production bonuses and half of the marketing support expenses, minus non-heaped renewal commissions of 5%. The administrative expenses include renewal commissions, overhead expenses and premium taxes, At quinquennial renewal years, issue expenses of 20% of the premium are assumed for policies qualified to reenter at select rates (25% for commissions, reduced by non-heaped renewal commissions of 5%).

4. Other Expenses

Expense studies indicate that underwriting and issue costs average $150 per policy and annual administrative costs $15 per policy in the size range assumed.

The valuation assumption is $150 per policy for issue expenses and an annual administrative expense of $20 ($5 for margin) inflated at 3% per year.

At quinquennial renewal years, an additional issue expense of $50 per policy (inflated at 3% per year from issue) is assumed for all policies.

SENSITIVITY TESTING

The following table shows the Policy Premium terminal reserves for the base case scenario using the assumptions described above.

BASE CASE (Policy Premium Method)

ISSUE	DURATION						
AGE	1	2	5	10	15	20	25
25	0.29	0.78	2.77	6.13	10.58	16.87	26.25
35	3.23	4.43	8.97	16.85	27.54	42.60	57.35
45	7.89	10.39	20.62	41.30	64.39	79.37	80.81
55	11.76	15.82	32.68	61.96	98.11	0.00	0.00
AVG.	5.75	7.76	15.97	31.06	49.32	49.96	58.37

RE-ENTRY PROPORTION

Two alternative scenarios were tested:

a) The proportion of lives, select at the last renewal date, who re-qualify at the next renewal date grades from 99% at age 25 to 75% at age 65 (instead of to 85% in the base case).

b) The proportion is always 100%

The results for the weighted average are:

PROPORTION OF SELECT LIVES AT LAST RENEWAL DATE WHO QUALIFY AT	DURATION						
NEXT RENEWAL DATE	1	2	5	10	15	20	25
99 TO 85% (BASE)	5.75	7.76	15.97	31.06	49.32	49.96	58.37
99 TO 75%	4.47	6.18	12.70	25.03	40.17	39.91	44.80
Level 100%	7.32	9.72	20.01	38.71	60.14	64.69	76.92

Grading the ultimate proportion to 75% instead of to 85% reduces reserves by about 20%. If the proportion is always 100%, reserves increase by about 25% at the early durations and by about 30% at later durations.

The choice of this assumption is difficult because of the lack of experience data. Unlike many assumptions, actual experience will depend upon the policy of the company towards renewal underwriting. Although the company has stated that it will strictly enforce its renewal underwriting policies, the valuation actuary recognizes that there will be pressure from brokers to lower the standards. The actuary is also aware that other companies have not rigidly enforced their renewal underwriting policies.

The valuation actuary decides that he should allow for some weakening of renewal underwriting practices, but not to the extent that has occurred in other companies. Although the proportions described in the baseline scenario are much higher than the proportion of new applicants who would qualify for standard rates, the actuary decides that it is appropriate to hold this margin. No change is therefore made to the baseline assumption.

THE FINAL VALUATION

The following 1978 Canadian Method mean reserves were calculated using the baseline Re-entry Proportion assumptions described above.

	DURATION						
ISSUE AGE	0.5	1.5	4.5	9.5	14.5	19.5	24.5
25	0.48	0.90	2.54	5.32	8.84	13.77	20.96
35	2.89	4.05	7.40	13.65	21.86	33.25	44.32
45	7.04	9.49	16.06	31.59	48.51	61.42	66.45
55	10.94	14.97	24.70	46.50	73.93	28.67	0.00
AVG.	5.26	7.24	12.48	23.94	37.67	36.89	46.78

VALUATION METHOD

Reserves are calculated using similar methods and formulae as for Single Scale 5. The re-entry plan is valued as a term to age 75 plan with premium rates at quinquennial renewal dates determined by the following formula:

$$GP[t] = [RP \times GPS] + [(1-RP) \times GPU],$$

where ***RP*** = Re-entry Proportion
GPS = Select (preferred) renewal rate
GPU = Ultimate (guaranteed) renewal rate

The premiums are level for the 5 year periods starting at the quinquennial renewal dates. Premium-related expenses are also calculated in this way, using the same proportions. Commissions at re-newal dates are calculated similarly, recognizing that the commission rate may vary depending upon whether the policyholder re-enters at select or ultimate rates.

APPENDIX C

MORTALITY DETERIORATION

1. GENERAL DISCUSSION

Select mortality tables generally show rates increasing with duration for the same attained age. Those who meet normal insurance company underwriting standards are more healthy than the population as a whole, but over time their health deteriorates, and they lose their advantage. But the deterioration in mortality is more rapid than could be accounted for by this fact alone. Those who lapse their policies, and thus are removed from the experience underlying the mortality table, tend to be more healthy on average than those who persist. It is reasonable to expect that the greater the lapse rate the more rapid the deterioration.

Renewable term insurance tends to have higher lapse rates than either permanent insurance or level-premium term insurance, especially at renewal dates. Many of those who lapse, perhaps most, are replacing their coverage with a policy from another company. They are able to determine expressly that they qualify for standard insurance. Of course, there will also be some unhealthy lives who lapse. The general rule holds for large groups of lives if not for all individuals in the group.

How is the rapidity of deterioration related to the lapse rate? To answer this question it is necessary to divide the lapse into three mutually exclusive components.

They are:

1. "Underlying" lapses are the part of the lapses comparable to what was experienced in the exposure underlying the construction of the select mortality table. The underlying lapse rate may be different for the pricing table and the valuation table. Most published tables are based primarily on permanent insurance of an era when ultimate lapse rates rarely exceeded 4%.

2. "Average" lapses are the part of the additional lapses which will exhibit mortality experience identical to that expected for the group of lives who persisted at least to the beginning of the current policy year.

3. "Selective" lapses are the remaining part of the additional lapses which will exhibit mortality identical to that of newly selected lives.

Since the mortality of all the lapses taken together is not likely to be superior to those of the third group nor inferior to those of the second group, all three components will be positive or zero.

It may seem more logical to divide the additional lapses into those who are selected and those who cannot qualify as select lives, but this is of no help in predicting the mortality of the persisting lives since we know nothing of the expected mortality of those who are unhealthy. Using the division given above, it is possible to develop a recursive formula which defines the expected mortality of the persisting group of lives.

2. FORMULA FOR MORTALITY DETERIORATION

In order to come up with a workable formula, it is necessary to make an idealizing assumption. That is, that the average and selective lapses occur only at policy anniversaries. Thus the average and selective lapse rates are applied to the population persisting just prior to the anniversary and acted on instantaneously at the anniversary. This presents no great hardship for the actuary since average and selective lapses will normally be assumed to occur only at renewal dates.

The underlying lapse rate, like the mortality rate, is assumed to apply continuously. Consequently the total lapse rate for a policy year will not be simply the sum of the three component lapse rates.

However, since at renewals the underlying lapse rate will almost always be quite small compared to the average and selective lapse rates, and since it is not likely possible to predict the average and selective lapse rates with a high degree of accuracy, likely harm will be done by approximating the total lapse rate as the sum of the three component lapse rates.

In the following development, the underlying lapse rate is assumed to continue to apply to those who have departed as average and selective lapses. In the experience underlying the construction of the select mortality table average and selective lapses would not have occurred. Hence the underlying lapse rate can be assumed to continue to apply to the (now hypothetical) persisting population in an environment in which the underlying lapse rate is the only lapse rate.

It is important to note that the following formula relies entirely on a trichotomy of lapse rates that cannot in fact be made. No experience studies on renewable term will ever yield underlying, average and selective lapse rates. Fortunately reasonable numbers are not too hard to develop. The underlying lapse rates, if not known, can often be determined or can be estimated fairly accurately. The additional lapses will show up in lapse studies; most companies will have statistically significant data. Mortality deterioration can be adjusted in future valuations as lapse experience emerges. The main element of judgement lies in subdividing the additional lapse into average and selective.

In the following formulas:

s is the duration at which the average and selective lapses occur

$q_{[x]+t}$ represents mortality rates from the standard select mortality table

$q'_{[x]+t}$ represents mortality rates form the table appropriate to the group of lives persisting to duration s, but just prior to the average and selective lapses. There may have already been some mortality deterioration prior to duration **s**. For the sake of determining this table, there is assumed to be no average or selective lapse rates at duration s or at any later duration. The mortality rates of this table are identical to those of the above table until the first selective lapses occur.

$q''_{[x]+t}$ represents mortality rates from the table appropriate to the persisting lives after the average and selective lapses at duration **s**,, assuming no further average or selective lapses. Mortality rates prior to duration **s**, are taken from the above table.

$q^{u}_{[x]+t}$ represents the underlying lapse rate applicable to policy year **t**

$q^{a}_{[x]+t}$ represents the average lapse rate which acts at exact duration **t**

$q^{s}_{[x]+t}$ represents the selective lapse rate acting at exact duration **t**. (Note that the average lapse rate and the selective lapse rate must be 0 for duration 0.)

The probability that a life in force just prior to the average and selective lapses at duration **s** will die between durations **s + t** and **s + t + 1** is:

$$\left[\prod_{r=0}^{t-1}\left(1-q'_{[x]+s+r}\right)\left(1-q^{u}_{[x]+s+r}\right)\right] q'_{[x]+s+t}\left(1-\frac{1}{2}q^{u}_{[x]+s+t}\right)$$

The probability that one of these lives will be an average lapse at duration s and then die in the above year is:

$$q^{a}_{[x]+s}\left[\prod_{r=0}^{t-1}(1-q'_{[x]+s+r})(1-q^{u}_{[x]+s+r})\right]q'_{[x]+s+t}\,(1-\tfrac{1}{2}q^{u}_{[x]+s+t})$$

The probability that one of these lives will be a selective lapse at duration s and then die in the above year is:

$$q^{s}_{[x]+s}\left[\prod_{r=0}^{t-1}(1-q_{[x+s]+r})(1-q^{u}_{[x]+s+r})\right]q_{[x+s]+t}\,(1-\tfrac{1}{2}q^{u}_{[x]+s+t})$$

The probability that one of these lives will persist until dying in the above year is:

$$(1-q^{a}_{[x]+s}-q^{s}_{[x]+s})\left[\prod_{r=0}^{t-1}(1-q''_{[x]+s+r})(1-q^{u}_{[x]+s+r})\right]q''_{[x]+s+t}(1-\tfrac{1}{2}q^{u}_{[x]+s+t})$$

But this last probability is by definition equal to the first probability less the sum of the second and third probabilities. Therefore, eliminating the factors in the underlying lapse rate which are obviously common.

$$(1-q^{a}_{[x]+s}-q^{s}_{[x]+s})\left[\prod_{r=0}^{t-1}(1-q''_{[x]+s+r})\right]q''_{[x]+s+t}$$

$$=(1-q^{a}_{[x]+s})\left[\prod_{r=0}^{t-1}(1-q'_{[x]+s+r})\right]q'_{[x]+s+t}-q^{s}_{[x]+s}\left[\prod_{r=0}^{t-1}(1-q_{[x+s]+r})\right]q_{[x+s]+r}$$

$$\therefore\ {}_{t|}q''_{[x]+s}=\frac{(1-q^{a}_{[x]+s})\ {}_{t|}q'_{[x]+s}-q^{s}_{[x]+s}\ {}_{t|}q_{[x+s]}}{1-q^{a}_{[x]+s}-q^{s}_{[x]+s}}$$

Assuming the select period is 15 years, the above mortality table will follow the underlying ultimate beginning 15 years after the last selective lapses. A Valuation Actuary who considers the mortality deterioration to run off too rapidly, can construct an underlying table with a longer select period. It would not be proper to use a select period of less than 15 year, particularly for the higher ages.

3. APL PROGRAM

The following APL program uses the above formula to calculate mortality rates applicable to persisting lives.

```
     ∇ Q←L MORDE X;W1;W2;W3;W4;I;J
[1]   ⍝ Calculate the result of mortality deterioration by selective lapses
[2]   ⍝ on a standard 15-year select mortality table
[3]   ⍝ X: ρ=2: risk class code, issue age
[4]   ⍝ L: A matrix of lapse rates.  The first row is for average lapse rates,
[5]   ⍝    the second for selective.  The first column applies to duration 0;
[6]   ⍝    there is one column for each mortality rate to be calculated.
[7]   ⍝    Some ofthe lapse rates may be 0.  L[;1] is not used.
[8]   ⍝ Q: Mortality rates applicable to persisting policyholders.
[9]   ⍝    (ρQ)=(ρL)[2]
[10]  ⍝ SELQ: =4 71 15: Select mortality for 4 risk classes, issue age 0 to
[11]  ⍝    70, durations 0 to 14.  Note that there cannot be a selective
[12]  ⍝    after the highest issue age of the table.
[13]  ⍝ ULTQ: =4 106: Ultimate mortality for 4 risk classes, attained ages
[14]  ⍝     0 to 105.
[15]  ⍝
[16]  ⍝ construct a matrix of standard select mortality rates for issue
[17]  ⍝ ages x[2] to the highest age at wich selective lapses occur
[18]   W4←⍳+/∨\⌽L[2;]≠0
[19]   W1←SELQ[X[1];X[2]+W4;⍳15⌊(ρL)[2]]
[20]   I←⍳ρQ←(ρL)[2]↑SELQ[X[1];X[2]+1;],(X[2]+15)↓ULTQ[X[1];]
[21]  ⍝ Apply the effect of selective lapses each year as they occur
[22]  L1:→(0=ρI←1↓I)/0
[23]   →(L[2;I[1]]=0)/L1
[24]   J←(15⌊ρI)↑I
[25]  ⍝ Calculate deferred probabilities of mortality
[26]   W2←Q[J]×1,×\1-Q[¯1↓J]
[27]   W3←W1[J[1];⍳ρJ]×1,×\1-W1[J[1];⍳¯1+ρJ]
[28]   W4←((W2×1-L[1;J[1]])-W3×L[2;J[1]])÷1-+/L[;J[1]]
[29]  ⍝ Convert deferred probabilities to normal annual mortality rates
[30]   Q[J]←W4÷1-0,+\¯1↓W4
[31]   →L1
     ∇
```