

Memorandum

To: All Fellows, Affiliates, Associates, and Correspondents of the Canadian Institute of Actuaries and other interested parties

From: Conrad Ferguson, Chair
Actuarial Standards Board
Dean Stamp, Chair
Designated Group, Equity Model Calibration Criteria

Date: July 28, 2017

Subject: **Final Communication of a Promulgation of Calibration Criteria for Equity Investment Returns Referenced in the Standards of Practice for the Valuation of Insurance Contract Liabilities: Life and Health (Accident and Sickness) Insurance (Subsection 2370)**

Document 217080

Introduction

According to subsection 2370 of the Standards of Practice:

Stochastic scenarios

.02 Where investment returns are stochastically modelled, the calibration of stochastic models used in the valuation should meet the criteria for investment returns as promulgated from time to time by the Actuarial Standards Board. [Effective April 15, 2017]

At its meeting on July 27, 2017, the Actuarial Standards Board (ASB) decided to promulgate the use of the calibration criteria for equity returns described below, effective October 28, 2017. The process being used to implement this is described in section E of the ASB's Policy on Due Process for the Adoption of Standards of Practice.

As mentioned in subsection 2370 of the Standards of Practice, calibration of investment returns would be promulgated from time to time. The calibration criteria promulgated in this document are expected to be reviewed approximately every five years.

An [initial communication](#) regarding this promulgation was published on May 2, 2017, with a comment period ending on June 30, 2017.

Rationale

The Standards of Practice outline a minimum insurance contract liability basis with respect to the model calibration, and reference prescribed criteria for investment returns.

The change to the promulgated calibration criteria for equity returns is being proposed for the following reasons:

1. To reflect more recent market experience. The current equity return calibration criteria are based on data from January 1956 to June 2010, while the proposed calibration criteria are based on data from January 1956 to December 2015; and
2. To reflect the expansion of hedging programs in recent years. Hedging programs rely on volatility criteria, which create a requirement to provide additional guidance regarding equity volatility model calibration.

A research paper, [Calibration of Equity Returns and Volatility for Stochastic Models](#), was released concurrently with the initial communication by the Canadian Institute of Actuaries (CIA) Committee on Life Insurance Financial Reporting (CLIFR); it provides a rationale for this promulgation for calibration criteria for equity returns and volatility. This promulgation applies to equity returns and volatility only.

The methodology and data start period used to update the equity return calibration criteria are consistent with that used to develop the existing equity return calibration criteria.

Volatility criteria are now being promulgated, and were designed to be consistent with the accumulation factor criteria, with the use of percentiles and projection horizons. The criteria were established with restrictions on ensuring a minimum level of volatility with the expectation that reserves would be sensitive to high volatility. Criteria were not established on low, median, or average volatility as we did not want to dictate specific model forms or best estimate assumptions.

The calibration criteria promulgated in this document are intended to apply to the calibration of all stochastic models used for the determination of insurance contract liabilities, where such models require assumptions regarding real-world equity returns and volatility. The calibration criteria are not intended to apply to risk-neutral assumptions.

An advisory note was published by the Office of the Superintendent of Financial Institutions (OSFI) in December 2010 setting out calibration criteria for investment returns applicable when calculating capital requirements for segregated funds with an internal model. The actuary is reminded that OSFI criteria apply to the calculation of capital requirements for segregated funds only, and that the criteria set out in this document apply to the calculation of any insurance contract liabilities using stochastic modelling of equity returns. However, nothing prevents the actuary from satisfying the OSFI criteria when those criteria are more stringent than those set out in this document.

Promulgation

The criteria are applied by assessing whether the scenarios that are generated by the model and used for valuation meet the various metrics as defined herein. If a closed-form formula

exists for a statistic subject to the criteria, it is sufficient to test that the theoretical value of the statistic calculated using the closed-form formula meets the criteria, as long as a large number of scenarios are used for valuation, and the actuary tests that the discrepancy between the theoretical value and the value calculated with the scenario set is not material.

The actuary is reminded that the promulgated calibration criteria are lower and upper bounds for various statistics, and that the actuary is expected to use sound statistical techniques and up-to-date data to parameterize the model used in valuation. This process could result in more conservative statistics than the promulgated criteria.

To model the investment returns of a specific fund, a proxy for the fund would be constructed. The proxy usually takes the form of a linear combination of market indices. The criteria described below apply to the investment returns generated for equity indices that are used in the composition of the proxies. All calibration criteria apply to returns in local currency.

References are made below to indices of developed non-Asian economies (L1 indices). The developed non-Asian economies are those contained in the following table:

Developed Non-Asian Economies			
Americas	Europe and Middle East		Pacific
Canada	Austria	Italy	Australia
United States	Belgium	Netherlands	New Zealand
	Denmark	Norway	
	Finland	Portugal	
	France	Spain	
	Germany	Sweden	
	Ireland	Switzerland	
	Israel	United Kingdom	

Recommended Criteria

The following table presents the maximum values for the accumulation factors for broad-based equity indices of developed non-Asian economies (L1 indices) and small capitalization equity indices and economies (L2 indices):

Left tail calibration criteria	1 year			5 years			10 years			20 years		
	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th	2.5th	5th	10th
L1 indices	0.74	0.81	0.88	0.70	0.80	0.95	0.80	0.95	1.20	1.25	1.65	2.25
L2 indices	0.68	0.76	0.85	0.60	0.70	0.90	0.70	0.90	1.20	1.10	1.55	2.35

In addition, the range for the expectation (mean) of the one-year horizon accumulation factor is as follows.

Mean Criteria	Min	Max
L1 indices	8%	12%
L2 indices	11%	15%

Calibration criteria have also been developed for the volatility of equity returns, as shown below. Consistent with accumulation factor and return calibration criteria, two sets of calibration criteria on the volatility of equity returns have been established: one applicable to

broad-based equity indices of developed non-Asian economies (L1 indices), and one applicable to small capitalization equity indices and economies (L2 indices). The following table presents the minimum values for the realized volatility of equity returns:

Right-tail Volatility Calibration Criteria	1 year		5 years	
	90th	95th	90th	95th
L1	21.50%	24.60%	19.10%	20.50%
L2	29.00%	32.60%	25.00%	26.50%

The above criteria are to be applied to the annualized standard deviation of continuously compounded monthly returns over the first one-year and first five-year projection horizons.

The criteria are to be applied by working through the decision tree described below. The guiding principle here is to use data where such are available and credible.

The decision tree is as follows:

- Case 1: If a large proportion of the index is comprised of a diversified basket of L1 indices, of an L1 index of a large economy, or of L2 indices, then the relevant set of calibration criteria applies to this index.
- Case 2: If the index does not fall under case 1, but the actuary has sufficient credible data about returns for the index in question, then the process has three steps:
 - a. Perform a model test. The model would first be fitted to the S&P TSX Composite total returns from January 1956 to December 2015. The model outputs are then compared to the equity return and volatility calibration criteria for L1 indices. If the model outputs satisfy those criteria, then the form of the model is acceptable and the actuary can proceed to the second step. If not, then the actuary would change the model form.
 - b. Fit the model to the available data for the index. The model is then used to generate returns.
 - c. A final test is to review the Sharpe ratio¹ of the model outputs. The Sharpe ratio is to be calculated using the expectation and the standard deviation of the one-year accumulation factor. The Sharpe ratio would not exceed 0.40 with an assumed risk-free rate of 4.00%. If necessary, the fitted parameters for the mean from step b. would be adjusted downward until this Sharpe criterion is satisfied.
- Case 3: If the index does not fall under case 1 or case 2, then
 - a. The equity criteria to be applied are derived from criteria for the L1 indices with an adjustment for the expected differences in mean returns and volatility.
 - b. The volatility criteria to be applied are derived from criteria for the L1 indices with an adjustment for the expected differences in the average and variability of

¹ The Sharpe ratio of an index is equal to the difference between the expected return of the index and the risk-free rate, divided by the standard deviation of the index.

volatility.

At a minimum, the index would be no less volatile than the TSX. If appropriate, the assumed volatility would be adjusted upward to reflect the stated objectives of the index. Details regarding both of these adjustments are provided in the appendix.

Comments Received

The DG has received no comments following the publication of the initial communication.

Criteria for the Adoption of Standards of Practice

The ASB's Policy on Due Process for the Adoption of Standards of Practice was followed in the development of the promulgation. The equity return calibration criteria promulgation meets the criteria set out in section B of the ASB's Policy on Due Process for the Adoption of Standards of Practice.

1. It advances the public interest through the use of a consistent basis for establishing equity return models for segregated fund business.
2. It provides for the appropriate application of professional judgment within a reasonable range. The proposed calibration criteria allow the actuary to use any model that fits with the promulgated criteria for equity return and volatility.
3. Use of the proposed decision tree and tables is practical for actuaries with relevant training. The calibration does not require use of a specific model, only that the scenarios used fit the calibration criteria.
4. The specified decision tree and tables are considered to be unambiguous.

Effective Date

The calibration criteria would be used for valuations on or after October 28, 2017, and early implementation in 2017 is permitted.

Members of the Designated Group

The members of the designated group are Craig Fyfe, Ricardo Mitchell, Stephan Sabourin, Dean Stamp (Chair), Na Ta, David Tian, and Pierre-Alexandre Veilleux.

CF, DS

Appendix: Application of Case 3 Criteria

The criteria for the accumulation factor of the index are the following:

$AF(F, p, t) = AF(TSX, p, t) \times \exp(\mu_{Diff} \times t + \sigma_{Diff} \times \Phi^{-1}(p) \times \sqrt{t})$ where,

$AF(F, p, t)$ is the left tail criterion for index F for the p^{th} percentile at horizon t ;

$AF(TSX, p, t)$ is the left tail criterion for L1 indices for the p^{th} percentile at horizon t ;

$\Phi^{-1}(p)$ is the inverse cumulative distribution function of the standard normal distribution;

σ_{TSX} is the sample standard deviation for the TSX;

σ_F is the sample standard deviation for the index;

σ_{Diff} is equal to $\sigma_F - \sigma_{TSX}$, the differential in the standard deviation of the two indices;

μ_{TSX} is the sample mean for the TSX;

μ_F is the mean for the index, calculated using the Sharpe ratio as

$$\mu_F = r + \sigma_F \times (\mu_{TSX} - r) / \sigma_{TSX};$$

μ_{Diff} is equal to $\mu_F - \mu_{TSX}$, the differential in the mean of the two indices; and

r is the risk-free rate established at 4% for the calibration.

The sample mean and the sample volatilities are calculated based on historical monthly log returns. The sample volatilities for the TSX and the index would be calculated using the longest common historical period available with the end date of December 2015. The sample mean for the TSX would be calculated using the data from January 1956 to December 2015.

At a minimum, the index would be no less volatile than the TSX. If appropriate, the assumed volatility would be adjusted upward to reflect the stated objectives of the index.

The volatility criteria to be applied are derived from criteria **for the L1 indices** with an adjustment for the expected differences in **the average and variability of volatility**.

$$V(F, p, t) = V(TSX, p, t) \times \exp \left\{ \left(\mu_F^{(logV)} - \mu_{TSX}^{(logV)} \right) \times t + \left(\sigma_F^{(logV)} - \sigma_{TSX}^{(logV)} \right) \times \Phi^{-1}(p) \times \sqrt{t} \right\}$$

where,

- $V(F, p, t)$ is the right tail criterion for index F for the p^{th} percentile at horizon t ;
- $V(TSX, p, t)$ is the right tail criterion for L1 indices for the p^{th} percentile at horizon t ;
- $\Phi^{-1}(p)$ is the inverse cumulative distribution function of the standard normal distribution;
- $\mu_F^{(logV)}$ is the sample mean of the non-overlapping one-year log-volatilities for index F ;
- $\sigma_F^{(logV)}$ is the sample standard deviation of the non-overlapping one-year log-volatilities for index F ;
- $\mu_{TSX}^{(logV)}$ is the sample mean of the non-overlapping one-year log-volatilities for the TSX;
- $\sigma_{TSX}^{(logV)}$ is the sample standard deviation of the non-overlapping one-year log-volatilities for the TSX;

The non-overlapping one-year log-volatilities are the natural logarithm of the sample standard deviations of non-overlapping one-year monthly log returns obtained by starting at the end of the data period and working backwards. The sample standard deviations for the TSX and the index would be calculated using the longest common historical period available with the end date of December 2015.