Research Paper

Considerations on the Quantification of Variability in P&C Insurance Policy Liabilities

Committee on Risk Management and Capital Requirements

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Memorandum

To: All Property and Casualty Insurance Practitioners

From: Phil Rivard, Chair
Practice Council

Michel Dionne, Vice-Chair
Committee on Risk Management and Capital Requirements

Date: November 7, 2012

Subject: **Research Paper – Considerations on the Quantification of Variability in P&C Insurance Policy Liabilities**

The purpose of this research paper is to provide education for the actuary when evaluating the variability surrounding estimates of policy liabilities, both claim liabilities and premium liabilities. This paper is directed at property and casualty (P&C) insurance only. Economic capital (EC) is defined as the amount of capital that a company is required to hold to ensure its solvency over the time horizon under consideration at a specified probability level. In order to calculate EC the development of stochastic models is required. A statistical framework for the analysis of variability surrounding estimates of policy liabilities will provide more information about the capital needs by line of business and/or coverage. In turn, this could help provide a better focus on enterprise risk management (ERM). The techniques discussed in this paper can also be used to evaluate a stochastically-derived provision for adverse deviation (PfAD) of the policy liabilities.

In accordance with the Institute's Policy on Due Process for the Adoption of Guidance Material Other than Standards of Practice, this research paper has been approved by the Committee on Risk Management and Capital Requirements, and has received approval for distribution from the Practice Council on September 7, 2012.

If you have any questions or comments regarding this research paper, please contact Michel Dionne, Vice-chair, Committee on Risk Management and Capital Requirements, at his CIA Online Directory address, michel.dionne@intact.net.

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1. INTRODUCTION

1.1 Purpose and Scope

The primary purpose of this research paper is to provide education regarding the numerous considerations for the actuary related to the analysis of variability surrounding estimates of claims costs in the calculation of policy liabilities for property and casualty (P&C) insurance. Policy liabilities include both the valuation of claims liabilities and premium liabilities. At the same time, the paper provides an introduction to the topic of economic capital (EC) for P&C actuaries.

It is important to recognize that this is not a paper on EC or EC modelling, although these topics are introduced and frequently referred to. Instead, this paper is primarily directed at the variability in the estimates of policy liabilities. This variability is a critical component of EC modelling. The measurement of this variability can also be important for the derivation of margins for adverse deviation (MfAD), the analysis of reinsurance, the pricing of insurance products, and enterprise risk management (ERM)—the process designed to identify and manage all potential risks across the enterprise and, if the risk does exist, minimize its impact on the organization.

The paper has been written with a focus on P&C insurance. While the theories could be applied to short-term liabilities of many forms of life insurance, that is outside the scope of this paper. The paper also excludes analysis of P&C catastrophe risk, such as earthquakes. Catastrophe risk is a critical and complex component of EC modelling of insurance risk and deserves its own research paper.

This research paper relies on publications of the International Actuarial Association (IAA), the Casualty Actuarial Society (CAS), and the Property and Casualty Compensation Insurance Compensation Corporation (PACCIC). Specifically, the CIA was granted permission by all three organizations to use the following text books and research paper without the requirement for formal notation of direct quotes:

- *Stochastic Modeling – Theory and Reality from an Actuarial Perspective*, 2010, Ottawa, Canada, available from the IAA for purchase at www.actuaries.org;

- *Estimating Unpaid Claims Using Basic Techniques*, 2009, Jacqueline Friedland, available from the CAS at www.casact.org/pubs/Friedland_estimating.pdf;

- Casualty Actuarial Society Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves, issued in May 1988, available from the CAS at www.casact.org/standards/princip; and

- Research Paper on Time Horizons and Terminal Provisions, 2010, available from PACICC at http://tinyurl.com/8a23mgh.

1.2 Terminology – Methods and Models

On September 8, 2011, the Actuarial Standards Board (ASB) released the Notice of Intent Regarding Standards of Practice for Modeling – A New Section Added to the General Section of the Standards of Practice. The ASB noted that the creation and use of models is inherent to actuarial work. In its notice of intent, the ASB proposed the following definition for a model:

A model is a mathematical representation of some aspect of the world that is based on simplifying assumptions.

P&C actuaries frequently use the terms *model*, *methodology* (or *method*), *technique*, and *approach* interchangeably, particularly in the quantification and evaluation of policy liabilities. In this paper, we do not differentiate between these terms.

1.3 Potential Use of Stochastic Techniques

The stochastic techniques discussed in this paper can be used:

- To determine provisions for adverse deviations (PfAD) for claims liabilities;
- To determine PfAD for premium liabilities;
- As input to EC models;
- To evaluate the range of profitability for pricing exercises; and
- For other strategic decision making, such as reinsurance purchases.

1.4 Current Guidelines for Regulatory Capital and MfADs Related to Policy Liabilities

Under current guidelines for required capital of the Office of the Superintendent of Financial Institutions Canada (OSFI), the minimum capital test (MCT) applies a percentage on claim liabilities between 5% to 15% and a percentage on premium liabilities of 8%. Although this was regarded as a major improvement from a flat premium-to-surplus ratio, a more robust statistical framework around policy liabilities has the potential to provide more information about the capital needs of P&C insurers. This is particularly true when considering the variability that could arise for insurers operating in different regions of the country, with different lines of business, and with different reinsurance programs.

Currently, the vast majority of Canadian P&C insurers judgmentally select MfAD by line of business[1] for claim liabilities and premium liabilities. Until December 2009, the Standards of Practice of the CIA set out a range for claims development MfADs of a low of 2.5% and a high of 15%. Effective December 31, 2009, the Standards of Practice were revised and now specify a high margin, selected for deterministic analyses, of 20%.[2]

The CIA December 2009 educational note Margins for Adverse Deviations for Property and Casualty Insurance addressed the change in the high margin from 15% to 20%:

> The change in the high margin from 15% to 20% was not intended to shift all selected margins for P&C insurers. Many actuaries currently select between 10% and 15% for many of the longer-tail lines of P&C insurance. These claims development margins are selected based on a review of the numerous considerations underlying the actuary's estimate of claim liabilities and premium liabilities. It is not expected that these margins would change simply due to the increase in the high margin. However, if there has been a notable change in the

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[1] References to line of business in this paper should be interpreted to be lines of business (also referred to as coverages) or sub-lines (or sub-coverages) within lines of business. For example, some actuaries may analyze general liability separately by bodily injury and property damage and may select different MfAD by these coverages. Similarly, automobile accident benefits may be analyzed separately for: medical benefits, income replacement, funeral, and death benefits.

[2] CIA, Standards of Practice, paragraph 2260.02.

environment and in the actuary's assessment of the various considerations that influence the selection of the margin for adverse deviations, then a change may be justified.

1.5 Complexity of Quantifying Uncertainty in Policy Liabilities

Quantifying the uncertainty associated with policy liabilities is complex. One approach is the use of simulation-based methodologies to determine the best estimate of policy liabilities (either claim liabilities, premium liabilities, or both) and the variability around such best estimate values. Variability, calculated in this manner, is typically expressed in terms of confidence intervals or conditional tail expectation (CTE)—the average amount of loss given that the loss exceeds a specified quantile. Another approach is the use of statistical methods, with or without simulation analysis, to express the variability of policy liabilities. This approach might show the mean of the distribution, variance, higher moments, or various percentiles. A third approach is to fit empirical distributions of claims to mathematical curves. Finally, relying on deterministic approaches and using alternative methods or assumptions (which may be deterministic, stochastic, or judgmental) may help to express the variability of policy liabilities.

At this time, there is no consensus in the industry, either in Canada or globally, about the best approaches for quantifying the variability of P&C policy liabilities. Nevertheless, actuaries endeavour to better understand the distributions that underlie both the claim liabilities and premium liabilities so that they can better assess the underlying risk of the company.

This research paper is separated into 10 sections and two appendices:

1. Introduction
2. Brief Introduction to Economic Capital
3. Types of Risk to be Considered
4. Data Issues
5. Statistical Considerations
6. Prior Risk
7. Current Rrisk
8. Model Considerations
9. Evaluating the Results
10. Conclusion
    Appendix A: Bibliography
    Appendix B: Examples.

2. BRIEF INTRODUCTION TO ECONOMIC CAPITAL

2.1 The Concept of Economic Capital

Risk measurement focuses on unexpected events. For P&C insurers, unexpected losses often arise either through having to pay more than anticipated for liabilities or achieving lower than expected returns from assets. It is unexpected losses that can lead to volatility in the earnings of an insurer—ranging from lower profits to balance sheet losses and, potentially, bankruptcy.

The concept of economic capital (EC) links the amount of capital required to ensure an insurer's continued solvency with the risk that the insurer undertakes. To achieve this link between capital and risk, the different risks are typically measured individually and aggregated to a single risk

metric, both by business line and across the insurer as a whole. Even in the case of rare events that might generate unusually high unexpected losses (e.g., natural catastrophe) the insurer strives to have sufficient capital to ensure the viability of the institution.

In the specific case of insurance companies, risks are highly diverse and different business activities can lead to various unexpected events. Examples of such risks include:

- Natural catastrophes (insurance risk—concentration risk);
- Dramatic changes in the value of market instruments (market risk);
- Inadequate pricing which results in premiums that are insufficient to cover claims (insurance risk—pricing);
- Inadequate processes resulting in incorrect collection of insurance premiums (operational risk); and
- Inadequate estimate of policy liabilities (insurance risk—reserve).

These types of events have occurred with some frequency in the past, and since they can significantly decrease the value of the insurer and the owners' capital, insurers would attempt to measure, manage, and control risk-taking based on their business objectives. Consequently, capital fulfills the purposes of shielding the insurer against unexpectedly high realizations of these risks and, potentially, bankruptcy.

The unpredictability of claims can lead to volatile financial results. To address these issues, insurers commonly create buffers in the form of general provisions for claims that might be reasonably expected to occur (i.e., policy liabilities). However, actual claims are often different from expectations, and capital is held to cover unforeseen possibilities.

2.2 Economic Capital Defined

In the literature, one finds many definitions of EC. The Basel Committee on Banking Supervision defines EC as "a measure of the amount of capital that a firm believes is needed to support its business activities or set of risks . . ."[3]. EC can be considered as a buffer against potential unexpected losses to a pre-defined solvency standard over a given time. Moreover, EC is often regarded as "the capital required to ensure a specified probability (level of confidence) that the firm can achieve a specified objective over a given time horizon."[4] EC can also be defined as the amount of capital an insurer needs to hold (i.e., shareholders have to invest) given its entire risk and return structure in order to limit the probability of bankruptcy over a specified time horizon. The key concept underlying all these descriptions of EC is the link between capital and risk.

EC is often referred to as risk-based capital because of its relationship to risk measurement. EC links capital to the level of risk within an organization and the measurement of that risk. It is in the measurement of risk that stochastic methodologies become necessary.

A key benefit of EC management is enhanced business performance as a result of allocating scarce capital resources to value-enhancing operations. By measuring risk versus return in a

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[3] Basel Committee on Banking Supervision Joint Forum. Trends in Risk Integration and Aggregation. August 2003, page 21.
[4] Goldfarb, Richard. CAS Exam 8 Study Note: Risk-Adjusted Performance Measurement for P&C Insurers, page 5. August 2006 (revised December 2006).

consistent manner throughout the organization, management is able to make strategic decisions on retaining, growing, or shrinking businesses. EC helps organizations develop tailor-made planning and budgeting tools.

2.3 Economic Capital Modelling and Enterprise Risk Management

EC modelling is an integral part of a well-defined ERM and corporate governance structure within an organization. From a functional perspective, ERM can contribute to an increase in the insurer's value, primarily by reducing the volatility in cash flows, preserving the financial stability of the insurer, and preventing the requirement to raise expensive additional funding. In this sense, EC measurement is a key tool for the risk management function in understanding and quantifying the risk undertaken so as to support capital adequacy and value-based management.

It is the opinion of many that EC modelling and ERM are completely interlinked. One cannot have a successful EC modelling process without ERM, and similarly meaningful ERM within an organization requires some form of EC modelling. Most sophisticated solvency regimes (like Basel II and Solvency II) require banks and insurers, respectively, to integrate ERM with their EC modelling process.

2.4 Identification of Material Risks

In an EC planning framework, an insurer would include all material risks. Material risks usually include those identified in the following figure.[5]
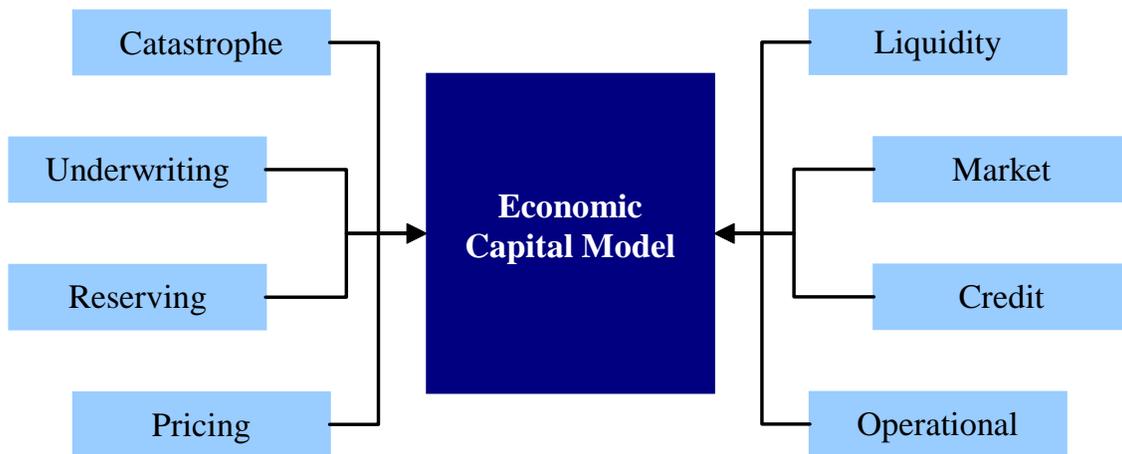
One can find many differences in the categorization of risks in international literature on EC and insurer solvency. Various references for this information can be found in the bibliography at the end of this research paper.

Regardless of how the risks are categorized, the important point is that insurers strive to identify and evaluate all relevant material risks within the organization's EC modelling framework.

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[5] Note on Enterprise Risk Management for Capital and Solvency Purposes in the Insurance Industry, March 31, 2009, IAA, page 52.

2.5 Variability of Estimates of Policy Liabilities

The purpose of this research paper is to begin to address the topic of the variability of estimates of policy liabilities within the CIA literature. Such variability, which is often referred to as reserving risk, is typically categorized as a component of insurance risk.

In Canada, the financial impact of previously written insurance policies is split between the estimated cost of fulfilling the unexpired portion of the insurance contracts (premium liabilities) and the estimated value of the unpaid claims (and related settlement costs) for claims that have already incurred as of the balance sheet date (claim liabilities). While the best estimate of the historical claim liability can be a huge amount, it is only shown as a single value in financial statements. Furthermore, the value of premium liabilities is not shown explicitly in the financial statements.

In order to use policy liabilities in an EC model, it is necessary to estimate distributions of possible outcomes for both claim liabilities and premium liabilities.

The process of estimating the variability of historical claims also allows actuaries to estimate the variability of future claims and cash flows, and from this analysis, actuaries are able to quantify major components of insurance risk, such as:

- Unpaid claims, or reserving risk—for claims that have already occurred;
- Pricing risk—for claims that will occur in upcoming financial periods; and
- Cash flow risk—for the loss of investment income when actual cash outflows exceed expectations.

The variations in the projected cash flows can also be used to analyze one of the major financial risks—asset/liability matching risk—or, more precisely, the effects of mismatching cash inflows and outflows.

It is important for the actuary to clearly understand what is included within each type of risk category. Some may define reserving risk to include both expired and unexpired policies, while others may include risk associated with unexpired policies with pricing risk. More important than any specific categorization is that the actuary ensures that all components of insurance risk are considered and incorporated into the EC model.

Quantitative analyses of these risks result in distributions of possible outcomes and/or simulated output that can be used within a larger EC model.

3. TYPES OF RISK TO BE CONSIDERED

3.1 Prior Risk and Current Risk Categories

Risk associated with the estimation of policy liabilities can be subdivided into the following categories:

- *Prior risk* (or claims liability risk) is the risk that the best estimate of claims liabilities for claims incurred up to the valuation date will be insufficient.
- *Current risk* (or premium liability risk) is the risk that the best estimate of future claims associated with the unearned premiums will be insufficient. A complete analysis of the topic of current risk would include consideration of catastrophe risk. It is important to

recognize that discussion and analysis of catastrophe risk is outside the scope of this paper.[6]

3.2 Source of Risks in the Estimation of Policy Liabilities

The uncertainty associated with the estimation of policy liabilities arises from the following three sources: process risk, parameter risk, and model risk.

Process risk is the fundamental uncertainty that is due to the presence of randomness, even when all other aspects of the distribution are known. Process risk includes the random variation inherent in the underlying claims development process. It refers to the risk of claims volumes differing from the mean. For example, even if the mean frequency and the mean severity are properly evaluated, the actual observed results (i.e., claims counts and dollars) will generally vary from the underlying means. Process risk is often considered relatively unimportant for portfolios of substantial size.

Parameter risk is the uncertainty that arises due to unknown parameters of statistical models for the distribution, even if the selection of those models is perfectly correct. Parameters of the chosen distribution may be misestimated as a result of statistical estimation errors or a shift in parameters over time as a result of changing environmental circumstances. Parameter risk includes the risk that the historical experience was, in fact, not the expected experience of the past.

Model risk, also known as specification risk, is the uncertainty that arises if the specified distributions or underlying models are unknown; in other words, the risk that the mathematical model of the process is inappropriate. This type of risk has two parts. First, there is the risk that the model is not a good representation of the current claims process. This risk can be estimated or reduced by testing the assumptions of the model(s) being used and studying the results from different models. Second, there is the risk that, even if the model is a good representation of the current claims process, this process will experience unforeseeable changes in the future. Examples include the emergence of asbestos claims and the financial crisis of 2008–2009. Model or specification uncertainty is frequently the largest single source of variability in estimates of policy liabilities and often the source most difficult to quantify.

It is important for the actuary to consider each of these categories of uncertainty in an analysis of prior risk (claim liabilities risk) and current risk (premium liabilities risk).

4. DATA ISSUES

This section addresses key data issues for the actuary's consideration that are related to the evaluation of policy liabilities, both claim liabilities and premium liabilities. The following topics are discussed:

- Homogeneity and credibility;
- Use of external data;
- Changes in the environment;
- Number of years of experience to include in analyses; and
- Outliers.

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[6] For more information concerning catastrophe modelling, the reader is invited to consult the Insurance Bureau of Canada paper Catastrophe Modelling Best Practices for Canadian Insurers and Reinsurers. 2011.

4.1 Homogeneity and Credibility

The process for estimating policy liabilities is often improved by subdividing experience into groups exhibiting similar characteristics, such as comparable claims experience patterns, settlement patterns, or size of loss distributions. For a heterogeneous product, such as commercial multi-peril (CMP) or general liability (GL) insurance, consideration may be given to segregating the experience into more homogeneous groupings, such as property and liability coverage for CMP insurance or bodily injury and property damage coverage for GL insurance. Other examples include separately analyzing the experience of personal and commercial risks and primary and excess limits. Additionally, subdividing or combining the data to minimize the potentially distorting effects of operational or procedural changes would be fully explored.

Credibility is a measure of the predictive value of a body of data. The degree of consideration given to homogeneity is related to the consideration of credibility. Increasing the homogeneity of the group of data or increasing the volume of data in the group tends to increase credibility. If the actuary divides the data into too many homogeneous groupings, however, there is a risk that the volume of data in the individual groups may become insufficient to perform a reliable analysis.

Obtaining homogeneous groupings requires refinement and partitioning of the total database. There is a point at which partitioning divides data into cells with too little volume to provide credible parameters. Each situation requires a balancing of homogeneity and the credibility of the data in each grouping. There may be different considerations for credibility and homogeneity for the estimation of claim liabilities and premium liabilities. Line and coverage definitions suitable for estimating policy liabilities for large insurers can be in much finer detail than for small insurers. Where a very small group of claims is involved, use of external information such as industry experience may be necessary.

In EC models, the partitioning of the data into more homogenous groups requires the estimation of the correlation between the different groups. This topic is addressed in more detail in section 8.2.

4.2 Use of External Data

It is important that actuaries recognize the potential shortcomings in the use of data generated from external sources. Entity-specific data is preferred over external data due to the risk that external data may be misleading or irrelevant due to differences relating to: insurance products, case outstanding and settlement practices, insurer's operations, coding, geographic areas, and mix of business and product types. Thus, the actuary would carefully evaluate the relevance and value of external data and use it only where internal data is not sufficiently credible.

4.3 Changes in the Environment

Changes that occur, both internally at the organization as well as externally in the economic, regulatory, and legal environments, can influence the data and thus the methodologies selected for estimating policy liabilities.

The installation of a new information technology (IT) system, an accounting change, a reorganization of claims responsibility, the introduction of a call centre, or changes in claims handling practices or underwriting programs are examples of operational changes that can affect the continuity of the claims experience. Similarly, changes in contract provisions, such as policy limits, deductibles, or coverage attachment points, may alter the amounts of claims of an insurer. Such contractual changes may affect both the frequency and severity of claims.

External influences include the judicial environment, regulatory and legislative changes, residual or involuntary market mechanisms, and economic variables such as inflation.

Abrupt changes, internally or externally, can be of particular concern to the actuary. For example, the introduction of no-fault auto insurance clearly changes the business environment. Government may implement legislation which affects insurance premiums or changes limits on claims for specific injury types; such changes can be implemented with prospective or retrospective implications to insurers. Such actions could result in significant changes in an insurer's policy liabilities. Following a legislative reform, the actuary may need to adjust historical data or select alternative estimation methods if there is a high likelihood that future development patterns may not follow historical patterns. Canadian automobile insurance has seen tremendous reforms in numerous provinces over the past few decades. Thus, there is an expectation that there may continue to be reforms in the future and such possibility might be considered in quantifying the variability surrounding estimates of policy liabilities.

Similar to the considerations of homogeneity and credibility, operational and environmental changes may affect the analysis of claim liabilities to a different extent than the analysis of premium liabilities. Recent government reforms or court decisions may have a direct impact on premium liabilities since the premium collected is generally a fixed amount.

However, when evaluating the variability in the estimate of the claim costs, one needs to be very careful when adjusting historical data, which may alter the historical volatility of the data.

4.4 Number of Years of Experience to Include in the Analyses

The actuary seeks to obtain sufficient data so that estimates of policy liabilities are reliable. The need for sufficient data does not necessarily mean that an actuary relies upon a large number of years of data since the actuary will also consider the effect of operational and external factors such as shifts in the mix of business, inflationary factors, legal and regulatory change, and changes in claims handling practices. If the book of business or the insured risks change significantly, the actuary may consider whether or not using fewer years of data increases the predictive power of the data set. When there are substantial changes over time, older years of data may be given less credibility than more recent years of data.

4.5 Outliers

When developing estimates of policy liabilities, actuaries frequently identify outliers in the data. Outliers can refer to either unusually large or small claim values, either claim counts or severities, or claims ratios (i.e., claims divided by earned premiums). The inclusion of outliers within the data set may have a significant influence on the results of various methodologies for projecting estimates of policy liabilities and the variability surrounding such estimates. Thus, actuaries often consider alternatives as to the treatment of such outliers.

Outliers may be tempered (i.e., removed or adjusted). This tempering would be done once a proper investigation has been conducted to ensure that the outlier is truly an anomaly or system error, and not something systematic in the experience. The actuary may decide to adjust or remove rogue data points in the historical experience when estimating policy liabilities.

It is important to recognize that retaining outliers may be important when trying to measure the variability of the policy liabilities. Removing unusually high or low data points may artificially reduce the observed variability, and thus artificially reduce statistical measurements such as confidence intervals and CTE. When measuring variability, the actuary would strive to exclude

only those data points that are incorrect due to system errors or where the risk variability is included in an independently derived catastrophe component of the analysis (e.g., the 1998 ice storm), such that there is no overlap between the provisions. Exclusion of catastrophe events would typically decrease the historical volatility and could lead to an understatement of the variability in estimates of policy liabilities. However, the actuary may exclude catastrophe events if the risk associated with such events is reflected in an explicit catastrophe risk, and therefore the actuary would seek not to double-count the catastrophe risk when measuring historical variability of claims experience.

5. STATISTICAL CONSIDERATIONS

As noted in the Introduction section, one outcome of this research paper is to provide P&C actuaries with an introduction to the topic of EC, while the primary purpose is to provide education about considerations surrounding the variability in estimates of policy liabilities. It is impossible to address all relevant issues in one research paper. Both EC and the variability surrounding policy liability estimates are rapidly evolving areas with new developments frequently emerging.

In this section, we address three factors that determine the capital requirement of an insurer: time horizon, terminal provision, and measurement of risk (or probability).

5.1 Time Horizon

The time horizon refers to the fixed time period used in EC models for the determination of capital requirements and/or the testing of the financial condition of an organization.

In January 2010, OSFI released a document titled Guidance for the Development of a Models-Based Solvency Framework for Canadian Life Insurance Companies (OSFI Life Guidance). In this guidance document, OSFI defines time horizon as "the length of the time period over which an initial shock is assumed to occur and capital is to provide protection for the identified risks." In this context, time horizon refers to the period over which a shock is applied to the risk and the period over which that shock will affect the insurer. At the end of the shock period, it is expected that capital would be sufficient so that assets meet the liabilities which have been recalculated to take into account the shock. The recalculation of the liabilities would allow for the effect of the shock on the liabilities over the full time horizon of the policy obligations.

According to the CIA draft paper Risk Based Economic Capital – Time Horizon, there are four general considerations for selecting the time horizon:

- Delays in acting. The time horizon would be long enough to reflect delays that will arise between the calculation date and the point at which a company or regulator could take action.

- Solvency confidence level. Longer time horizons require lower confidence levels, but it is not always easy to choose a level of confidence for a longer period that is consistent with a shorter period confidence level.

- Long-term risks/trends. As the insurer's obligations may extend significantly beyond the time horizon, the insurer may be exposed to some risks that transpire gradually over time. Therefore, it is important that where the time horizon is shorter than the term of the liabilities, allowance is made for these future obligations via the terminal liability measure used at the end of the time horizon.

- Model error and other practical considerations. The longer the time horizon, the more significant the impact of model risk becomes. With longer horizons it also becomes much more difficult to predict the impact of changes in management or policyholder behaviour. Using longer time horizons can also increase the complexity and run-time of the cash flow models used in the EC projections, particularly when an assessment of the solvency position is required at intermediate points in the time horizon.[7]

Risk Based Economic Capital – Time Horizon identifies four different approaches for time horizon:

- One year with a terminal provision;
- Multi-year with a terminal provision;
- Run-off with periodic solvency testing; and
- Run-off without periodic solvency testing.

There continues to be tremendous debate within the global actuarial community as to whether a one-year or multi-year time horizon is most appropriate for EC modelling purposes. The actuary would consider the purpose(s) of the EC model and evaluate the implications of the different alternatives when selecting the time horizon. Different considerations and opinions are presented in the PACICC research paper which is cited in the bibliography of this paper.

5.2 Terminal Provisions

In OSFI Life Guidance, the terminal provision is defined as "the valuation basis for the risk that remains at the end of the initial time period." In Actuarial Aspects of Internal Models for Solvency II, the authors offer two alternatives for the consideration of terminal provisions:

- Run-off: the terminal value represents the cost of settling insurer obligations as they come due. This terminal value depends on the distribution of ultimate settlement values for policy obligation foreseen as run off commences.
- Transfer Value: the terminal value is determined from a model representing the cost of transferring the unsettled policy obligations to another entity at the end of the time horizon or projection period. This terminal value depends on two distributions. Firstly, it depends on the distribution of the estimates of ultimate settlement values for policy obligations at the end of the projection period. Secondly, it depends on the distribution of market risk perception, as measured by changes in cost of capital or other measures. (The one-year runoff of the Solvency Capital Requirement (SCR) calibration of Solvency II is a transfer value test.)[8]

The above two issues are also often discussed in terms of a choice between a going concern approach and a transfer approach. In the going concern approach, the objective is to ensure that the insurer has sufficient assets at the end of the time horizon to ensure that all remaining obligations can be met given experience at a conservative confidence level. The terminal provision on a going concern basis depends on the distribution of ultimate settlement values for policy obligations foreseen as runoff commences. As noted above, the transfer approach requires

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[7] CIA Committee on Risk Management and Capital Requirements' Solvency Framework Subcommittee, draft dated November 4, 2005.
[8] Brooks et al, 2009.

a terminal provision consistent with the cost in the market to sell the business. The lack of a market for transfer values may be a problem with this alternative.

The CIA draft paper Economic Capital: Calculation of Terminal Provision identifies two key principles applicable to terminal provisions:

- The terminal provision must reflect experience along a given path and its resulting conditions at the end of the time horizon. Thus the terminal provision will generally be unique for each scenario within the time horizon.

- Second, the terminal provision must reflect the remaining obligations at the end of the time horizon over their entire remaining lifetime.[9]

The CIA Solvency Framework Subcommittee identifies five key issues surrounding the terminal provision:

- What will a company do following an adverse tail event year, e.g., continue in operation or transfer the remaining business to a third party? Knowing this dictates the function of the terminal provision and leads to a valuation method.

- Which general valuation method is appropriate, Historical Experience (Real World) or Market Consistent (Risk Neutral)?

- When should the terminal provision include margins, and how might they be calculated?

- How can risks that do not fully develop within the time horizon be captured in the terminal provision calculation?

- Are there appropriate approximations available to simplify the computational complexity implied by having to calculate a path-specific terminal provision at the end of every stochastic scenario over the time horizon?[10]

Further discussion on terminal provisions is out of scope of this document. Readers should refer to the various papers cited in this section.

5.3 Measure of Risk

An insurance portfolio may be thought of as a collection of insurance policies, including their associated assets and liabilities. Let a_B be the value of the assets and l_B be the value of the liabilities as posted in the Balance Sheet. Then we define Available Capital (AC) as $k \equiv a_B - l_B$. Let us define the random variable $X = (L - l_B) - (A - a_B)$, where L is the (random variable) value of all future cash flows related to claims and other expenses and A is the (random variable) value of all future cash flows of the associated assets (e.g., as purchased with premiums), the particular sample path values of which will only be known once the last claim has settled and all expenses have been paid. We define Required Capital (RC) as $\rho[X]$ where ρ is an appropriate risk measure focusing on values of X in the right hand tail of its distribution. For solvency, we have the solvency inequality:

$RC \leq AC$, or equivalently, $\rho[X] \leq k$

As long as any risk measure being used satisfies the translation invariance property discussed below, at any given point in time before this, we may take the value of X, conditional on

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[9] CIA Risk Management and Capital Requirement Committee's Solvency Framework Sub Committee, draft dated October 6, 2006.
[10] Ibid.

information known up to that time, as the difference between the value of the then-unpaid claims and other expenses and that of the then-current assets, by subtracting the paid-to-date claim and other expense costs from both sides, and we can therefore think of L as being the value of the liability for unpaid claims and other expense costs and A as the being the value of the current assets at any particular given time.

At any given point in time, we may estimate the value of X by means of an appropriate (e.g., fair value) measurement (for Balance Sheet purposes) of the L = the value of the liability for unpaid claims and expenses, and A = the value of the current supporting assets.

For EC purposes, we are interested only in the behaviour of the random variable X in its right-hand tail. In the context of an insurance portfolio, required capital may be thought of as the prudent amount of capital needed to ameliorate the risk of X being far out in the right-hand tail of its distribution. Note that for this paper, we focus on insurance risk and ignore variability in the value of the assets.

If, at any given point in time, there are no assets supporting a particular insurance portfolio, and if the only liabilities are those in connection with the then-unpaid claims, then X = the excess of the value of the then-unpaid claim liability over its Balance Sheet posted value.

5.3.1 Desirable Properties of Risk Measures for Total Assets Requirement

A number of desirable properties for risk measures for required capital have been identified and discussed in the literature. Principal amongst these are the following properties:

Translation Invariance

A risk measure ρ is translation invariant if for any number λ, we have:

$$\rho[X + \lambda] = \rho[X] + \lambda$$

For example, in the special case where A is zero with probability one, this means that if the value of the liabilities is increased by a constant amount, the required capital will not change but the total asset requirement (l_B + RC) will increase by the same amount.

If ρ is translation invariant, then the solvency inequality becomes:

$$\rho\,[\,L - A\,] \leq 0$$

Homogeneity

A risk measure ρ is homogeneous if for any number $\lambda \geq 0$ we have:

$$\rho[\lambda \cdot X] = \lambda \cdot \rho[X]$$

For instance, if we choose to measure everything in thousands of $ rather than in $, then the required capital, as a number, would be only one thousandth of what it was when measured in $.

Monotonicity

A risk measure ρ is monotonic if whenever $X_1 \leq X_2$ with probability one, we have:

$$\rho[X_1] \leq \rho[X_2]$$

For instance, in the special case when A is zero with probability one, this means that if the excess of the value of the liabilities over their posted value in the balance sheet in one

portfolio is never more than that in another portfolio, then its required capital will not be more than that of the other portfolio.

Sub-Additivity

A risk measure ρ is sub-additive if for any X_1 and X_2 we have:

$$\rho[X_1 + X_2] \le \rho[X_1] + \rho[X_2]$$

This says that if we diversify by adding a second insurance portfolio, then the required EC for the combined portfolios would not exceed the sum of the required capital amounts for each of the underlying portfolios.

A risk measure which has all of the above four properties is called a coherent risk measure.

A risk measure which has the 2nd and 4th of the above properties is also a convex risk measure, that is, for all $0 < \lambda < 1$ and any X_1 and X_2 we have:

$$\rho[\lambda \cdot X_1 + (1 - \lambda) \cdot X_2] \le \lambda \cdot \rho[X_1] + (1 - \lambda) \cdot \rho[X_1]$$

5.3.2 Examples of Risk Measures for Economic Capital

In the following, the function G(x) is defined as the complement of F(x), the cumulative distribution function of the random variable X, that is:

$$G(x) \equiv P[X > x] \equiv 1 - F(x)$$

The following are a few examples of risk measures for EC:

Sufficient Number of Standard Deviations above the Mean

Where μ is the mean and σ is the standard deviation, for n > 0 this risk measure is defined as:

$$\rho[X] \equiv \mu_X + n \cdot \sigma_X$$

This is not a coherent risk measure because it is not monotonic.

This measure of risk for required capital has been criticized for the fact that, unless X has a distribution which is symmetric about its mean, then its standard deviation is affected by what the left-hand tail of the distribution looks like, while here, we are really only interested in the right-hand tail. This may be more of an issue when the range of X includes arbitrarily large negative values.

Value at Risk (VaR)

For $0 < q < 1$, this measure of risk is the $(1 - q)^{th}$ quintile of the distribution of X, defined as:

$$\rho[X] \equiv VaR_{1-q}[X] \equiv \inf\{x|G(x) < q\}$$

This measure of risk for required capital takes required capital as the smallest amount required to make the probability of (eventual) ruin (i.e., X > 0) tolerably small.

It is not a coherent risk measure because it is not sub-additive.

Tail Value at Risk (TVaR)

For $0 < q < 1$, this measure of risk is the average of all values of VaR in excess of VaR_{1-q}, defined as:

$$\rho[X] \equiv TVaR_{1-q}[X] \equiv (1/q) \cdot \int_0^q VaR_{1-y}[X]dy$$

Most practitioners define both the TVaR and the CTE to have this definition.

By looking at a graph of G(x) versus x, and integrating variously along the vertical axis or along the horizontal axis, one finds that:

$$TVaR_{1-q}[X] = VaR_{1-q}[X] + (1/q) \cdot \int_{VaR_{1-q}[X]}^{\infty} G(x)dx$$

This is a coherent risk.

For continuous random variables and $0 < q < 1$, this measure of risk can be equivalently defined as the conditional expectation of X out in the right-hand tail in excess of VaR:

$$\rho[X] \equiv E[X|X > VaR_{1-q}[X]]$$

By looking at a graph of G(x) versus x, and integrating along the horizontal axis, one finds that:

$$E[X|X > VaR_{1-q}[X]] = VaR_{1-q}[X] + (1/G(VaR_{1-q}[X]) \cdot \int_{VaR_{1-q}[X]}^{\infty} G(x)dx$$

This is not a coherent risk measure because it is not sub-additive, if X is not continuous.

From the above, we see that this measure and TVaR will be the same in certain circumstances, namely if:

$$q \equiv G(VaR_{1-q}[X])$$

Looking at a graph of G(x) versus x, one sees that this will be the case if and only if G(x) is a continuous function, or equivalently, X is a continuous random variable.

That is, for non-continuous random variables, the expectation definition may produce different results than the above definition of TVaR. We caution readers that the financial literature has not been consistent in defining TVaR and CTE. Readers may elsewhere find TVaR and/or CTE defined using this expectation definition. Simply understand that while the two definitions are equivalent for continuous random variables, they are not equivalent for non-continuous random variables and hence one should use this paper's definition of TVaR, not the expectation definition.

All required EC risk measures will measure the risk in whole currency units, and will therefore be discrete and not continuous random variables. However, for continuous random variable approximations to such discrete random variables, both forms of TVaR will be identical.

Care would be taken for situations where it may not be reasonable to approximate a discrete random variable with a continuous random variable for required EC purposes. For example, it may be inappropriate to try to approximate X with a continuous random variable if the insurance portfolio consists of a policy with no supporting assets where we know that there will be at most one single claim, and that such a claim, if it occurs, will occur at a fixed time and for a fixed amount, but with uncertain probability.

5.3.3 Numerical Example of VaR and TVaR

Assume a simulation of 1,000 loss amounts for an insurance portfolio, from a normal distribution with mean of $15,000 and standard deviation of $1,500. The insurer collects premiums of $18,000 from every insured. After ranking the outcomes the various risk measures, such as VaR and TVaR, can be computed. In this specific example, the 99% VaR is 593, the 98% VaR is 357, the 99% TVaR is 1,200 and the 98% TVaR is 817.

Table 3: Calculation of VaR and TVaR from simulation output

	Scenario #	Liability	Premium	Loss		
worst scenario	620	20,763	18,000	2,763		
	58	20,141	18,000	2,141		
	566	19,160	18,000	1,160		
	150	19,123	18,000	1,123		
	644	19,014	18,000	1,014		
	25	18,906	18,000	906	TVaR = Avg (1,200)	
	830	18,800	18,000	800		
	449	18,756	18,000	756		
	562	18,749	18,000	749		
	209	18,593	18,000	593	<--- 99% VaR	TVaR = Avg (817)
	159	18,575	18,000	575		
	592	18,544	18,000	544		
	205	18,510	18,000	510		
	140	18,439	18,000	439		
	362	18,413	18,000	413		
	411	18,401	18,000	401		
	141	18,370	18,000	370		
	456	18,366	18,000	366		
	3	18,360	18,000	360		
	321	18,357	18,000	357	<--- 98% VaR	
	327	18,327	18,000	327		
	…	…	…	…		
	975	11,130	18,000	-6,870		
	256	10,960	18,000	-7,040		
	918	10,813	18,000	-7,187		
best scenario	294	9,716	18,000	-8,284		

VaR and TVaR are heavily dependent on the parametric distribution inherent in the model. The selection of the distribution is an assumption of the model. Using simulation, one can obtain the simulations according to one or more parametric distributions. Using historical data, one would not be able to test whether the results are reasonable or if the model is properly calibrated because one could not possibly collect enough data to capture the 99th percentile.

One problem with the VaR measure is that it does not take into consideration what the claims will be if that worst-case event actually occurs. The claims distribution above the percentile does not affect the risk measure. TVaR is preferred to address some of the problems with the percentile risk measure.

6. PRIOR RISK – CLAIM LIABILITIES RISK

6.1 Introduction

Methodologies for analyzing the variability in prior risk can be classified as aggregate triangle-based methods and individual claims frequency-severity methods. The examples introduced in

this research paper are limited to a relatively short sample of aggregate triangle-based methods. For additional methods, including individual claims frequency-severity methods, we refer the reader to the following (which are all cited in the bibliography):

- IAA's textbook *Stochastic Modeling – Theory and Reality from an Actuarial Perspective*;
- Insurance Bureau of Canada's June 2012 paper titled Canadian Perspective on Models to Quantify Variability in P&C Insurance Policy Liabilities; and
- Richard Goldfarb's paper Risk-Adjusted Performance Measurement for P&C Insurers.

The specific type of methodology selected for measuring claim liabilities risk, which could then be used as a building block of an EC model, will depend on the types of data available, user familiarity, and available software, among other factors. As important as these criteria are for the selection of an approach, they are secondary to the way in which the output of the method will be used in the EC model.

At the end of this section, we present the results of selected aggregate methods applied to Alberta auto industry data. Various triangle-based methods based on total claims were tested; frequency-severity based methods were not considered.

6.2 Methods to Derive the Best Estimates of Claim Liabilities

Detailed discussion of the technology and applicability of current practices to develop the best estimate of claim liabilities is beyond the scope of this research paper. As mentioned previously, the primary focus is the variability surrounding estimates of policy liabilities. Nevertheless, the actuary would seek to avoid overlap between conservatism in the estimates of claim liabilities (and premium liabilities) and margins.

Indeed, there is a chance that the actuary's best estimate is not equal to the median or the mean values of the probabilistic distribution. In such circumstances, the actuary would determine the application of statistical measurements to the best estimate. For example, the actuary would determine if the confidence interval is based around the mean or around the actuary's best estimate of the claim liabilities.

6.3 Selected Aggregate Triangle-Based Methodologies

Several common methods used for determining a distribution around the estimate of unpaid claims are introduced below. Prior knowledge of statistical distributions and simulations will be valuable to the reader. The reader is also directed to the appendix for a detailed example of the calculations applied to industry data for Alberta auto using the Hertig, Mack, and Panning methods.

For further information on various models, please see the IAA textbook *Stochastic Modeling – Theory and Reality from an Actuarial Perspective*, cited in the bibliography of this paper. Another paper that gives a good summary of the methodologies is Risk-Adjusted Performance Measurement for P&C Insurers by Richard Goldfarb, which is cited in the bibliography to this paper.

6.3.1. Hertig (1983)[11]

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[11] Hertig, J. A Statistical Approach to IBNR-Reserves in Marine Reinsurance. 1985.

Hertig's methodology starts from the incurred chain ladder approach and uses the natural logarithm of the claims development factors to estimate the mean and variance by development period. Hertig then estimates a cumulative mean and variance from which it is possible to calculate different confidence intervals. The variability of recent accident years is very dependent on the variability of the tail of the triangle.

6.3.2. Mack (1993, 1999)[12]

Mack's method (1993) calculates the mean and standard errors of the chain ladder unpaid claim estimate. The error term incorporates both process and parameter variance.

The method is distribution-free. In order to calculate percentiles, a probability distribution is selected for the outstanding claim liabilities. As noted by Houltram (2003), Mack proposes a lognormal model. Li (2006) observes that other distributions can be used depending on the required thickness of the tail.

Mack's method is built around the chain ladder algorithm, and Venter (1996) notes that it should only be applied to claim processes that also follow the chain ladder assumptions. Consistent with the chain ladder framework, Mack makes the following three assumptions:

- Cumulative claims in each exposure period are independent;
- The expected value of cumulative claims for the next evaluation date (C_{j+1}) is equal to the cumulative claims at the current evaluation date (C_j) multiplied by the corresponding claims development factor; and
- The variance of a claims development factor is inversely proportional to the cumulative claims (at the evaluation to which the claims development factor will be applied).

This is an analytical method for estimating the standard error of the estimated liability based on the traditional chain ladder model for estimating ultimate claims. Its analytical tractability makes it ideal for the current purpose, where frequent stress testing of assumptions and methods is required, despite some inherent weaknesses of the method. The Mack method, as with most statistically-based methods, deals with a single statistical model, and in most cases considers only one data set (for example a paid claim development triangle). Therefore, the results would apply to the projections of a particular method and not necessarily to the final distribution of estimated liability.

The actuary would also be aware of what statistical element is being considered by a particular stochastic method. For example, does the distribution apply to expected forecasts for a method or to the forecasts themselves?

6.3.3. Hodes, Feldblum, Blumsohn[13]

Hodes, Feldblum, and Blumsohn describe an approach that is based on the chain ladder method and incorporates simulation. The approach involves simulating age-to-age claims development factors for each development period rather than relying on various averages.

[12] Mack, T. Measuring the Variability of Chain Ladder Reserve Estimates. 1993.
[13] Hodes, D. M, S. Feldblum, and G. Blumsohn. 'Workers Compensation Reserve Uncertainty'. *Proceedings* of the Casualty Actuarial Society, Volume LXXXVI: 263–392.

The approach is intuitively appealing to many actuaries and quite flexible. However, the intensive use of simulation, which is inherent in the methodology, could impact run-time.

6.3.4. Bootstrapping Method[14]

Bootstrapping is a versatile framework that can be used in combination with many other methods. The bootstrap premise is that there is an assumed model framework that provides a perfect "fitted" version of the data, and the difference between this fitted version and the actuarial historical data gives an indication of how different the actual data can be from the model framework. This is captured by calculating residuals from this difference. The residuals are placed in a "pool" that is sampled from and added to the fitted model to generate new versions of simulated "actual" data. For each simulated dataset, the model framework is applied to develop the claims to ultimate. If this process is simulated 10,000 times it will yield 10,000 values for the unpaid claims, forming an empirical estimate of the unpaid claims distribution.

The most common version of the bootstrap model is from England and Verrall (1999, with an update in 2002), using the chain ladder method.

The bootstrap framework inherits the assumptions of whatever underlying best estimate method is used. The bootstrap framework assumes that the best estimate methodology fits the data well and completely explains all past trends. If, for instance, the chain ladder method is used, and there are calendar year trends that have not been accounted for in the method, then the resulting residuals will not be independent and identically distributed.

6.3.5. Zehnwirth Method[15]

While the methods described previously attempt to adapt existing actuarial methods (e.g., chain ladder method) to produce estimates of the distribution, Zehnwirth proposed a different framework that relies on a ground-up probabilistic analysis of the claims development process. His approach works with the (log) incremental paid claims and identifies common trends impacting accident years, calendar years, and development periods simultaneously. Using an extensive probabilistic approach, Zehnworth produces estimates of the distribution of ultimate claims.

6.3.6. Panning

In Measuring Loss Reserve Uncertainty[16], Panning presents "a parametric method for measuring loss reserve uncertainty, specifically defined as the coefficient of variation of estimated future loss payments." Panning states that his proposed approach has four important virtues:

> First, it is <u>simple</u>, and so can be implemented on a spreadsheet and applied to universally available data. Second, the method is <u>accurate</u>, since it addresses and avoids a number of pitfalls in statistical estimation and also meets a Monte Carlo test of its precision. Third, the resulting estimates are <u>comparable</u> across different lines of business and different

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[14] England P. D., and R.J. Verrall. 'A Flexible Framework for Stochastic Claims Reserving'. *Proceedings* of the Casualty Actuarial Society, Volume LXXXVIII (2001): 1–38.

[15] Goldfarb, R. CAS Exam 8 Study Note: Risk-Adjusted Performance Measurement for P&C Insurers, December 2006.

[16] Panning, William H. Measuring Loss Reserve Uncertainty. Willis Re working paper, 2005.

firms. Finally, the measure of LRU [loss reserve uncertainty] is <u>scalable</u>, so that is applicable to reserves that have been estimated in different ways.[17]

Panning relies on development triangles of incremental paid claims and uses linear regression analysis applied separately to each accident year. He validated his results through a simulation approach which incorporated both process and parameter risk.

6.3.7. Stochastic Approaches

Monte Carlo simulation techniques can be applied to estimate distributions of claim liabilities. Simulation approaches can be applied to different variables such as age-to-age factors and expected claims ratios. Traditional methods (such as chain ladder, expected claims ratio, and Bornhuetter-Ferguson) can then be used with simulated values of these variables. The results of the stochastic analyses can be used to create distributions around the estimates of unpaid claims.

6.4 Test of Different Methodologies with Alberta Auto Data

Testing was conducted to evaluate the projected VaR and TVaR for different methodologies using Alberta automobile insurance data. The testing relied on Alberta private passenger (excluding farmers) all-industry development triangles as reported under the IBC Automobile Statistical Plan up to December 31, 2007. Several actuaries from different Canadian insurers were asked to provide key metrics based on this data following their usual methodologies. To the extent applicable, each actuary applied his or her own diversification methodology between the different coverages. Where possible, the actuaries calculated VaR and TVaR at the 90th, 95th, 99th, and 99.5[th] percentiles.

A summary of results follows. The average total estimate of unpaid claims over all accident years combined is $1,809,780,000. Table 1 presents the EC for claim liabilities at alternative percentile levels using a VaR percentile approach.

Table 1: Economic Capital at various percentiles for different methods

Economic Capital

Method	90.00%	95.00%	99.00%	99.50%
Hertig - Incurred	121 032	155 342	219 703	243 265
Panning - Intercept	89 656	115 072	162 749	180 203
Panning - Time factor	135 821	174 324	246 550	272 991
Mack Paid	154 713	198 572	280 844	310 962
Mack Incurred	80 631	103 489	146 367	162 063
Bootstrapping	118 803	143 938	249 354	249 470
Practical Stochastic	143 087	180 919	292 283	300 828
mean	120 535	153 094	228 264	245 683
std dev	27 262	34 853	55 849	56 786
CV	23%	23%	24%	23%

Table 2 shows the estimates above as a percentage of the average unpaid claims estimate, as well as a ranking of each estimate in comparison to the other methods. Graph 1 shows the same information as table 1, but in a graph format.

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[17] Ibid.

Table 2: Economic Capital charge and ranking of methods

Economic Capital					*ranking of results*			
Method	*90.0%*	*95.0%*	*99.0%*	*99.5%*	*90.0%*	*95.0%*	*99.0%*	*99.5%*
Hertig - Incurred	6.7%	8.6%	12.1%	13.4%	4	4	3	3
Panning - Intercept	5.0%	6.4%	9.0%	10.0%	2	2	2	2
Panning - Time factor	7.5%	9.6%	13.6%	15.1%	5	5	4	5
Mack Paid	8.5%	11.0%	15.5%	17.2%	7	7	6	7
Mack Incurred	4.5%	5.7%	8.1%	9.0%	1	1	1	1
Bootstrapping	6.6%	8.0%	13.8%	13.8%	3	3	5	4
Practical Stochastic	7.9%	10.0%	16.2%	16.6%	6	6	7	6

Graphic 1: Economic Capital Required
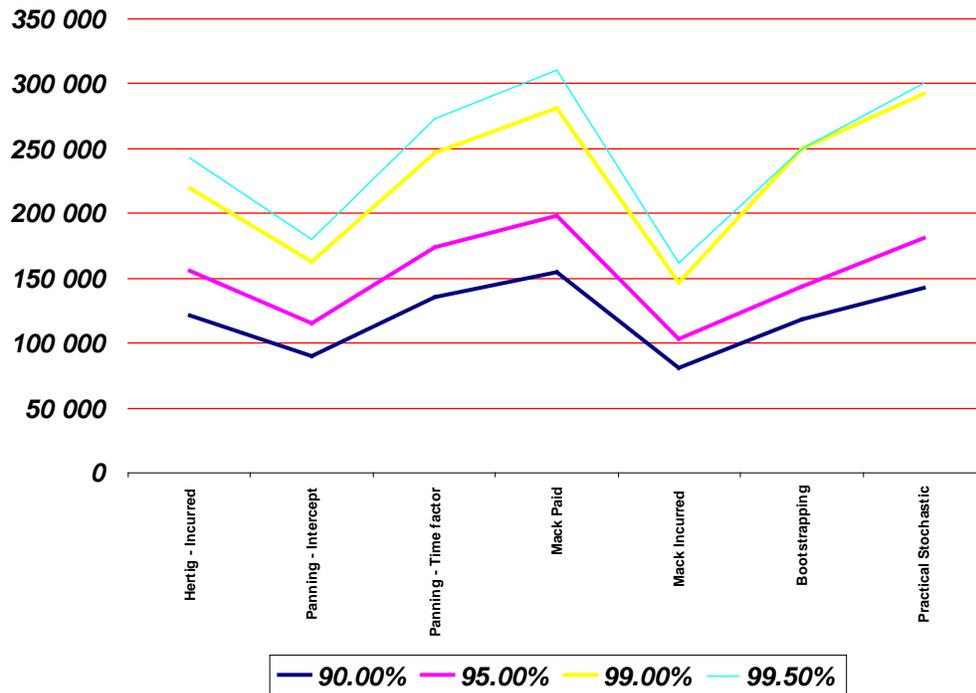
Appendix A presents more detailed examples of the calculations for the four methodologies: Hertig, Panning using the model with an intercept term, and Mack with both incurred and paid claims.

These examples were performed for Alberta personal auto third party liability bodily injury only.

7. CURRENT RISK – PREMIUM LIABILITIES RISK

While there has been extensive research and literature over the past 25 years on the topic of estimating claim liabilities and in particular the variability in estimates of claim liabilities, there are relatively few materials directed at premium liabilities. Some actuaries use frequency-/severity-based simulation methods to derive estimates of premium liabilities and thus have probability distributions from which to derive MfAD and input into EC modelling. The much more common approach in Canada, however, is the application of expected claims ratios to unearned premiums. These expected claims ratios are frequently selected based on a review of

historical experience with modifications for changes in rate level, retentions, inflation, and other environmental factors.

In 'Prediction Error of the Future Claims Component of Premium Liabilities under the Loss Ratio Approach'[18], Jackie Li presents an approach for analyzing the variability in premium liabilities. The abstract for this paper states:

> In this paper, we construct a stochastic model and derive approximation formulae to estimate the standard error of prediction under the loss ratio approach of assessing premium liabilities. We focus on the future claims component of premium liabilities and examine the weighted and simple average loss ratio estimators. The resulting mean square error of prediction contains the process error component and the estimation error component, in which the former refers to future claims variability while the latter refers to the uncertainty in parameter estimation. We illustrate the application of our model to public liability data and simulated data.[19]

Li concludes with the following comments:

> The formulae derived in this paper appear to serve as a good starting point for assessment of premium liability variability in practice. Nevertheless, there are practical considerations in dealing with premium liabilities such as the insurance cycle, claims development in the tail, catastrophes, superimposed inflation, multi-year policies, policy administration and claims handling expenses, future recoveries, future reinsurance costs, retrospectively rated policies, unclosed business, refund claims, and future changes in reinsurance, claims management, and underwriting. To deal with these issues, a practitioner needs to judgmentally adjust the data or make an explicit allowance, based on managerial, internal, and industry information.[20]

For the current risk, an actuary's analysis may focus on the extent that actual claims will differ from the expected claims underlying the premium liability estimate. Different statistical techniques can be used on claims ratios or loss costs of prior years to measure the variability of historical experience. Li's list of practical considerations will have implications to an analysis of either claims ratios or loss costs.

When using historical claim ratios, there is the introduction of the variability of premium adequacy within historical results. However, the effect of inflation over time would be minimized given steady growth in the underlying average premium. Also, the uncertainty about the premium adequacy in future results could mean it is reasonable to keep the relative premium adequacy in past data.

Using claim costs eliminates the issue of premium adequacy, but will introduce a potential bias for inflation, with a steady increase in the loss cost over time. The actuary would include adjustments to reflect the effect of inflation on the analysis.

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[18] *Variance*, 2010.
[19] Ibid.
[20] Ibid.

8. MODEL CONSIDERATIONS

8.1 Model Assumptions and Parameterization

The quality of any model depends on the quality of the assumptions. This point cannot be overemphasized because the results of any simulation model are only as good as the model used in the simulation process. If the model does not "fit" the data, then the results of the simulation may not be a good estimate of the distribution of possible outcomes.

Once the model selection process is completed, the data will then be analyzed and parameterized to ensure that the distributions of possible outcomes are reasonable. A variety of criteria can be used to test underlying model assumptions against the data and to test the model output to determine whether the model assumptions and results are reasonable. One of the key goals in this process is to make sure that each iterations of the model is a plausible outcome. If any outcome seems implausible, then the actuary would determine whether the outcome could be removed, i.e., constrained from the model, or if refitting the model would provide better results. Care is required not to remove improbable (plausible but unlikely) outcomes, because these tail events are a critical part of the distribution of possible outcomes.[21]

Model parameterization often involves multiple steps, because the actuary will generally want to use components of more than one model in an attempt to overcome the simplification inherent in any one model and to more accurately reflect the complex reality under analysis. Actuarial (and statistical) theory has long recognized that blending the best features of multiple models will generally result in a better overall model and helps to address model risk.

8.2 Aggregation of Risk and Correlation

It is widely recognized that the aggregate risk may be less than the sum of individual risks to the extent that these risks are not perfectly positively correlated with one another. In the following discussion, we restrict consideration to risk measures which are sub-additive, or else the aggregate risk may be greater than the sum of the individual risks, as may happen when applying some popular risk measures such as VaR. Typically, the insurance portfolios to which a P&C insurance company is exposed are not independent and have some interaction. Thus, the actuary will consider whether to explicitly reflect this lack of independence, or correlation.

When the EC has been calculated for separate components, the actuary will then determine the effect of correlation on the EC for the aggregate.

Correlation may reveal itself in a number of ways across different lines of business. For example, the cost of claims for multiple lines of business may be affected by inflationary forces or by contagion, such as that resulting from a new legal precedent or a catastrophic event. Correlation may also occur in incremental claim payments across lines of business. This type of correlation is relatively easy to measure and is illustrated in an example of the IAA text.

Correlation may occur among the relative adequacy level of premiums across lines of business. This phenomenon may be a manifestation of the underwriting cycle.

As with the estimation of any model parameter, the estimation of a measure for correlation is not an exact science. Thus, testing alternative correlation models (including key assumptions) would

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[21] For some models, the most likely implausible outcomes are negative amounts when only positive amounts are possible (e.g., no recoveries).

be prudent to gain insight into the sensitivity of such models. The actuary's choice as to how to address correlation will depend to a great extent on the lines of business analyzed, the model selected, and considerations related to credibility and homogeneity of the data.

In many cases, the correlations among risk factors are difficult to quantify due to limited historical data available for statistical analysis. This is a particularly sensitive problem in the tail of the distribution for two reasons. First, what happens in the tail is crucial to any reasonable measure of EC; and second, the correlation observed in the tail under extreme events may differ, perhaps substantially, from correlation in other parts of the distribution.

There are two primary approaches for the actuary to account for correlation: matrix of correlation coefficients and copula model.

The application of a matrix of correlation coefficients is relatively easy to implement, and the actuary can apply significant judgement to assess the plausibility of the selected correlation coefficients. However, the tail behaviour of the components, which is a crucial element in any reasonable EC risk measure, may not be appropriately captured by such correlation coefficients if the underlying correlation changes in the tail, where, of course, one may have little or no empirical data.

A model with copulas can be used. Sklar's theorem states that the joint cumulative distribution function of a multidimensional random variable may be expressed as a function C, the copula, of the cumulative distribution function of the marginal distributions along each dimension. This copula C is unique if all of these marginal distributions are continuous. The copula approach has the advantage of greater depth than correlation coefficients and reflects the effect of various general types of dependence. For example, the dependence among lines of business may have a different structure near the mean than out in the tail, where extreme events lie.

It is important to recognize that both matrices of correlation coefficients and copulas may suffer from insufficient data in the tail and thus lack of empirical support.

Other considerations include whether negative correlation between lines of business would be recognized in an EC model and how to take into account correlation between accident years or policy years and between prior risk and current risk.

As noted previously, the issue of correlation can also be related to homogeneity and credibility of the historical experience. Theoretically, any product grouping would lead to a consistent total risk evaluation. A perfect model would give identical results, notwithstanding the way the underlying products are mapped into groups. For example, one actuary might use a large number of small classes for correlation analysis. Each of these groupings will be more homogeneous than another mapping which used a small number of large classes. A consistency test could be performed to assess whether the correlation determination is robust. If this were so, the combined effect of individual risk measures and diversification benefits would show some consistency for various plausible mappings.

In addition to a diversification benefit across portfolios discussed below, within any portfolio, there might be a positive or negative benefit deriving from correlation between liability risks (such as pricing risk, underwriting risk, and claims risk) and asset risks (such as market risk, credit risk, and liquidity risk).

If one includes a diversification benefit arising because of less than perfect positive correlation between component insurance portfolios in the EC risk measure for the aggregate, then it

becomes a challenge to distribute the effect of this benefit to the component insurance portfolios. Such a diversification benefits may be applied to:

- Aggregates of different lines of business; and
- Aggregates of various entities of a worldwide group.

Once the diversification effects are calculated, the actuary would seek an estimate of the amount of EC the company could allocate to each of the components. Such allocation may be accomplished following a:

- Proportional approach in which each entity has the same percentage reduction in capital; and
- Marginal approach which takes into account the contribution of each of the entities to the total diversification.

Regardless of whether or not new International Financial Reporting Standards include or exclude consideration of diversification, actuaries would likely include the effect of diversification within EC models for purposes other than financial reporting.

9. EVALUATING THE RESULTS[22]

Once the statistical framework is built and the model is selected, the actuary would evaluate the adequacy of the resulting model. Such evaluation can be based on different approaches such as: sensitivity analysis, stress testing, and back testing.

In a sensitivity analysis, the actuary would consider the sensitivity of the unpaid claims estimates to reasonable alternative assumptions. When the actuary determines that the use of reasonable alternative assumptions could have a material effect on the estimates of policy liabilities, the actuary would notify management and attempt to discuss the anticipated effect of this sensitivity on the analysis with management.

In stress testing, the actuary would test extreme events. It has been recently observed that such events seem to occur more frequently than many models assume. Therefore, many actuaries routinely conduct stress testing whereby specific extreme movements are assumed and the changes in insurance portfolio value are observed. Examples of stress testing include significant shifts in the yield curve or discount rate and significant changes in the development patterns underlying the estimate of unpaid claims. Regular stress testing was adopted as part of OSFI's regulatory practices in 2010 for federally-regulated P&C insurers.

Back testing is another important type of test for the actuary to consider. In back testing, the actuary examines how well the model estimates would have performed in the past. For example, one could use part of the estimate of unpaid claims and calculate what would be the expected development at different percentiles near the best estimate. One expects that the more recent development would reasonably fit a pattern similar to the underlying distribution used in the model.

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[22] For further information concerning model implementation, validation and calibration, the reader is invited to consult the report Risk Assessment Models by the Solvency Framework Subcommittee Model Working Group of the Committee on Risk Management and Capital Requirements, August 2008.

10. CONCLUSION

Many techniques are available to the actuary to develop a statistical framework to quantify the variability associated with estimates of policy liabilities. As this is a rapidly developing area of actuarial science, there are few guidelines from either the actuarial professional organizations or insurance regulators. Judgment remains crucial in the analysis for both claim liabilities and premium liabilities.

There is no doubt that actuaries will continue to explore and integrate the use of stochastic approaches in their work for P&C insurance companies. This may be required in the future for financial reporting purposes, compliance with regulatory frameworks, and further development of ERM for P&C insurers, as well as the intrinsic value of such analyses in the management of P&C insurance companies. Actuaries will seek to expand their knowledge of and technical capabilities in this area. Further work is clearly required by the CIA to help support its membership in gaining such knowledge and technical skill. This research paper is intended simply to be the first step in such a process. For EC modelling and ERM purposes, further work is required to fully research and provide guidance on other key risks such as credit, market, catastrophe, and operational risk.

APPENDIX A: BIBLIOGRAPHY

Actuarial Standards Board. Actuarial Standard of Practice No. 43 Property/Casualty Unpaid Claim Estimates. 2007.

Basel Committee on Banking Supervision Joint Forum. Trends in Risk Integration and Aggregation. 2003.

Brooks, D., R. J. Care, M. B. Chaplin, A. M. Kaufman, K. A. Morgan, D. N. Roberts, J. M. E. Skinner, D. J. K Huntington-Thresher, P. J. Tuley, and D. L. Wong. Actuarial Aspects of Internal Models for Solvency II. Draft paper presented to the Institute of Actuaries. 2009.

Canadian Institute of Actuaries. Margins for Adverse Deviation for Property Casualty Insurance, Educational Note. 2009.

Canadian Institute of Actuaries Committee on Risk Management and Capital Requirements' Solvency Framework Subcommittee. Selection of Appropriate Risk Measures for Economic Capital. 2005.

———. Risk-Based Economic Capital – Time Horizon. Draft paper. 2005.

———. Economic Capital: Calculation of Terminal Provision. Draft paper 2006.

———. Risk Assessment Models. 2008.

Casualty Actuarial Society. Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves. May 1988.

Casualty Actuarial Society Dynamic Risk Modelling Handbook Working Party. *Dynamic Risk Modelling Handbook*.

Collings, S., and G. White. 'APRA Risk Margin Analysis'. Presented at the Institute of Actuaries of Australia XIIIth General Insurance Seminar. 2001.

England P. D., and R. J. Verrall. 'A Flexible Framework for Stochastic Claims Reserving'. *Proceedings* of the Casualty Actuarial Society, Volume LXXXVIII (2001): 1–38.

European Actuarial Consultative Group. Diversification. Technical paper. 2005.

Friedland, J. *Estimating Unpaid Claims Using Basic Techniques*. 2009.

———. Research paper on Catastrophe Modelling Best Practices for Canadian Insurers and Reinsurers. Insurance Bureau of Canada. 2011.

Goldfarb, R. Risk-Adjusted Performance Measurement for P&C Insurers. Casualty Actuarial Society Exam 8 study note. December 2006.

Hardy, M.R. An Introduction to Risk Measures for Actuarial Applications. Society of Actuaries study note. 2006: 1–30.

Hayne, R. M. 'Measurement of Reserve Variability'. Casualty Actuarial Society Forum (2003): 141–171.

Hertig, J. A Statistical Approach to IBNR-Reserves in Marine Reinsurance. 1985.

Hodes, D. M, S. Feldblum, and G. Blumsohn. 'Workers Compensation Reserve Uncertainty'. *Proceedings* of the Casualty Actuarial Society, Volume LXXXVI: 263–392.

Houltram, A. 'Reserving Judgement – Considerations Relevant to Insurance Liability Assessment Under GPS210', XIV General Insurance Seminar, Institute of Actuaries of Australia. 2003.

Insurance Bureau of Canada. Canadian Perspective on Models to Quantify Variability in P&C Insurance Policy Liabilities. 2012.

International Actuarial Association. Note on Enterprise Risk Management for Capital and Solvency Purposes in the Insurance Industry. 2009.

———. *Stochastic Modeling – Theory and Reality from an Actuarial Perspective.* 2010

Kirschner, G.S., Colin Kerley and Belinda Isaacs. 'Two Approaches to Calculating Correlated Reserve Indications Across Multiple Lines of Business'. *Variance*, Volume 2, Issue 1: 15–38.

Li, J. 'Prediction Error of the Future Claims Component of Premium Liabilities under the Loss Ratio Approach'.*Variance,* Volume 2, Issue 4: 155–169.

Mack, T. Measuring the Variability of Chain Ladder Reserve Estimates. 1993.

Mahler, H.C. 'An Example of Credibility and Shifting Risk Parameters'. *Proceedings* of the Casualty Actuarial Society: LXXVII (1990): 225–282.

Manistre, B. John. A Practical Concept of Tail Correlation. 2008.

Manktelow, B., E. Murphy and J. Friedland. Property and Casualty Insurance Compensation Corporation Research Paper on Time Horizons and Terminal Provisions. June 2010.

Panning, William H. Measuring Loss Reserve Uncertainty. Willis Re working paper. 2005.

APPENDIX B: EXAMPLES

The detailed calculations of four methods are presented in this appendix. The first is the Hertig method (incurred), followed by Panning's econometric method, and the Mack method (incurred and paid). All methodologies were performed on Canadian industry data for Alberta third party automobile liability bodily injury (BI) triangles from 1988 through 2007.

Hertig Method

The Hertig method is based on the claims development factors derived from cumulative incurred claims. Exhibit 1 and 2 display incurred claims and claims development factors, respectively.

Exhibit 1

Cumulative Incurred Claim Amounts ($000)

Acc. Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180 (...)	228	240
1988	79 766	112 165	145 420	167 211	176 562	175 260	177 188	179 155	178 544	178 437	177 634	178 023	177 555	177 536	177 537 (...)	177 538	177 538
1989	89 661	131 787	173 182	192 006	196 068	196 592	197 312	197 583	197 142	197 793	197 283	197 501	197 205	197 580	197 534 (...)	197 375	
1990	117 575	176 613	219 841	229 989	234 163	234 606	235 478	234 800	235 120	235 945	235 673	235 607	236 200	235 981	236 093 (...)		
1991	126 765	191 816	228 394	239 713	241 328	241 306	241 855	238 303	237 929	237 332	237 210	238 425	238 381	237 694	237 686 (...)		
1992	162 004	226 987	254 825	272 320	271 302	272 923	273 963	272 419	272 900	272 795	272 360	272 032	272 037	271 786	272 203 (...)		
1993	210 926	273 939	311 478	317 653	319 647	317 753	316 829	316 753	317 597	317 693	317 651	317 613	317 519	317 523	317 444		
1994	231 787	301 056	331 989	343 878	339 247	340 528	338 565	337 656	338 775	338 898	340 457	340 002	340 258	340 223			
1995	265 119	329 905	365 686	376 875	377 565	381 025	385 471	387 379	388 234	388 001	387 525	387 601	388 036				
1996	290 629	360 236	389 970	401 461	414 044	416 205	415 769	418 024	419 744	419 675	420 517	420 492					
1997	305 790	370 504	400 499	431 729	446 493	450 496	453 797	454 569	457 181	458 492	458 419						
1998	339 859	402 803	455 150	484 925	498 460	506 213	507 659	506 850	510 290	511 894							
1999	366 787	440 909	491 366	517 996	526 600	533 461	536 482	538 877	540 779								
2000	379 889	466 940	512 621	539 613	551 227	557 689	563 744	563 478									
2001	400 292	501 729	547 449	569 664	576 979	585 909	589 923										
2002	430 592	539 731	586 922	608 731	621 599	628 131											
2003	413 875	488 192	523 489	547 410	556 491												
2004	371 572	405 638	427 609	446 242													
2005	286 373	309 851	320 453														
2006	306 406	315 912															
2007	264 657																

Exhibit 2

Loss Development Factors

Acc. Year	Development Periods (months)																
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (…)	216-228	228-240
1988	1,4062	1,2965	1,1499	1,0559	0,9926	1,0110	1,0111	0,9966	0,9994	0,9955	1,0022	0,9974	0,9999	1,0000 (…)	1,0000	1,0000
1989	1,4698	1,3141	1,1087	1,0212	1,0027	1,0037	1,0014	0,9978	1,0033	0,9974	1,0011	0,9985	1,0019	0,9998 (…)	1,0000	
1990	1,5021	1,2448	1,0462	1,0181	1,0019	1,0037	0,9971	1,0014	1,0035	0,9988	0,9997	1,0025	0,9991	1,0005 (…)		
1991	1,5132	1,1907	1,0496	1,0067	0,9999	1,0023	0,9853	0,9984	0,9975	0,9995	1,0051	0,9998	0,9971	1,0000 (…)		
1992	1,4011	1,1226	1,0687	0,9963	1,0060	1,0038	0,9944	1,0018	0,9996	0,9984	0,9988	1,0000	0,9991	1,0015 (…)		
1993	1,2987	1,1370	1,0198	1,0063	0,9941	0,9971	0,9998	1,0027	1,0003	0,9999	0,9999	0,9997	1,0000	0,9998		
1994	1,2988	1,1027	1,0358	0,9865	1,0038	0,9942	0,9973	1,0033	1,0004	1,0046	0,9987	1,0008	0,9999			
1995	1,2444	1,1085	1,0306	1,0018	1,0092	1,0117	1,0050	1,0022	0,9994	0,9988	1,0002	1,0011				
1996	1,2395	1,0825	1,0295	1,0313	1,0052	0,9990	1,0054	1,0041	0,9998	1,0020	0,9999					
1997	1,2116	1,0810	1,0780	1,0342	1,0090	1,0073	1,0017	1,0057	1,0029	0,9998						
1998	1,1852	1,1300	1,0654	1,0279	1,0156	1,0029	0,9984	1,0068	1,0031							
1999	1,2021	1,1144	1,0542	1,0166	1,0130	1,0057	1,0045	1,0035								
2000	1,2292	1,0978	1,0527	1,0215	1,0117	1,0109	0,9995									
2001	1,2534	1,0911	1,0406	1,0128	1,0155	1,0068										
2002	1,2535	1,0874	1,0372	1,0211	1,0105											
2003	1,1796	1,0723	1,0457	1,0166												
2004	1,0917	1,0542	1,0436													
2005	1,0820	1,0342														
2006	1,0310															
2007																

The claims development factors (LDFs) are computed as the ratio of a development period to the prior period. For example, the 2006 12-24 factor of 1.0310 is equal to 315,912 / 306,406. Because a Lognormal distribution is assumed for incurred claims, the emphasis is put on the logarithmic of the development factors, which are shown in exhibit 3. Taking the same 2006 12-24 factor, then 0.0306 = LN(1.0310).

LDFs Logarithmic

Acc. Year	Development Periods (months)															
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (...)	216-228	228-240
1988	0,3409	0,2597	0,1396	0,0544	-0,0074	0,0109	0,0110	-0,0034	-0,0006	-0,0045	0,0022	-0,0026	-0,0001	0,0000 (...)	0,0000	0,0000
1989	0,3851	0,2732	0,1032	0,0209	0,0027	0,0037	0,0014	-0,0022	0,0033	-0,0026	0,0011	-0,0015	0,0019	-0,0002 (...)	0,0000	
1990	0,4069	0,2189	0,0451	0,0180	0,0019	0,0037	-0,0029	0,0014	0,0035	-0,0012	-0,0003	0,0025	-0,0009	0,0005 (...)		
1991	0,4142	0,1745	0,0484	0,0067	-0,0001	0,0023	-0,0148	-0,0016	-0,0025	-0,0005	0,0051	-0,0002	-0,0029	0,0000 (...)		
1992	0,3373	0,1157	0,0664	-0,0037	0,0060	0,0038	-0,0056	0,0018	-0,0004	-0,0016	-0,0012	0,0000	-0,0009	0,0015 (...)		
1993	0,2614	0,1284	0,0196	0,0063	-0,0059	-0,0029	-0,0002	0,0027	0,0003	-0,0001	-0,0001	-0,0003	0,0000	-0,0002		
1994	0,2615	0,0978	0,0352	-0,0136	0,0038	-0,0058	-0,0027	0,0033	0,0004	0,0046	-0,0013	0,0008	-0,0001			
1995	0,2186	0,1030	0,0301	0,0018	0,0091	0,0116	0,0049	0,0022	-0,0006	-0,0012	0,0002	0,0011				
1996	0,2147	0,0793	0,0290	0,0309	0,0052	-0,0010	0,0054	0,0041	-0,0002	0,0020	-0,0001					
1997	0,1920	0,0778	0,0751	0,0336	0,0089	0,0073	0,0017	0,0057	0,0029	-0,0002						
1998	0,1699	0,1222	0,0634	0,0275	0,0154	0,0029	-0,0016	0,0068	0,0031							
1999	0,1841	0,1084	0,0528	0,0165	0,0129	0,0056	0,0045	0,0035								
2000	0,2063	0,0933	0,0513	0,0213	0,0117	0,0108	-0,0005									
2001	0,2259	0,0872	0,0398	0,0128	0,0154	0,0068										
2002	0,2259	0,0838	0,0365	0,0209	0,0105											
2003	0,1651	0,0698	0,0447	0,0165												
2004	0,0877	0,0527	0,0427													
2005	0,0788	0,0336														
2006	0,0306															
2007																
Mean	0,2319	0,1211	0,0543	0,0169	0,0060	0,0043	0,0000	0,0020	0,0008	-0,0005	0,0006	0,0000	-0,0004	0,0002 (...)	0,0000	0,0000
Cum. Mean	0,4367	0,2047	0,0836	0,0293	0,0124	0,0064	0,0022	0,0021	0,0001	-0,0007	-0,0002	-0,0008	-0,0008	-0,0004 (...)	0,0000	0,0000
Variance	0,0117	0,0046	0,0009	0,0003	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000 (...)	0,0000	
Cum. Variance	0,0185	0,0062	0,0014	0,0004	0,0002	0,0001	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000 (...)	0,0000	
Distr. Mean	1,561880	1,230972	1,087967	1,030004	1,012582	1,006502	1,002207	1,002141	1,000116	0,999278	0,999802	0,999178	0,999203	0,999634 (...)	1,000000	1,000000
Distr. Variance	0,045494	0,009400	0,001607	0,000456	0,000160	0,000105	0,000076	0,000033	0,000023	0,000018	0,000012	0,000007	0,000004	0,000002 (...)	0,000000	0,000000
Cumul. Incurred	264657	315912	320453	446242	556491	628131	589923	563478	540779	511894	458419	420492	388036	340223 (...)	235697	197375
EC @ 90%	72343	39252	16465	12216	9031	8262	6581	4178	3316	2814	2009	1450	1048	631 (...)	0	0

Total Independance

EC @ 90%	86211

The lower part of the table shows the means and variances for each development period. Since these follow a Normal distribution, the cumulative mean is determined by:

$$\mu_i = \Sigma\ \mu_k \text{ for all } k = i$$

and cumulative variance, as per Hertig approximation, by:

$$\sigma^2_i = [\sigma^2_k * (n_k + 1)/n_k] + \sigma^2_{i-1} \text{ for } k = i$$

where *i* represents the cumulative development period and k represents the individual development period, zero being the latest one of the oldest accident year. And where *n* represent the number of observations in the kth development period.

Looking at the development period of 156-168, the individual mean μ_6 is (-0.0001 + … + -0.0001)/7 = -0.0004 and the cumulative means is -0.0004 - 0.0004 = -0.0008. The individual variance σ_6 is [(-0.0001 + 0.0004)^2 + … + (-0.0001 + 0.0004)^2]/(7 - 1) = 0.00000206, and the cumulative variance is 0.00000206 * (7 + 1)/7 + 0.00000210 = 0.00000445.

The Lognormal distribution mean and variance of the claims distribution can be computed as:

$\text{Mean}_i = e^{(\mu i + \sigma i 2/2)}$

$\text{Variance}_i = (e^{(2\mu i + \sigma i 2)})*(e^{\sigma i 2}-1)$;

Thus, for the same development period of 156-168, the mean_6 is $e^{(-.0008\ +\ 0.00000445/2)} = 0.999203$, while the variance_6 is $(0.999203)^2 * [e^{(0.00000445)} - 1] = 0.00000444$.

The EC values are derived at a given confidence level (90% in this case) using the normal approximation and the variance_i:

$\text{EC}_i = \text{CIL}_i * z_\alpha * v(\text{Variance}_i)$;

where CIL_i is the cumulative incurred claims for the ith development period and z_α is the Normal random variable value given the confidence level α. So, for the same period of development, the EC_6 would be $388,036 * 1.282 * (0.00000444)^{0.5} = 1,048$.

Finally, aggregation assumes independence between the development periods. This validation can be tested using the incurred Mack model. It can yield a low but non-zero correlation between the accident years. Given the results by line of business, the Hertig method can be adjusted by the resulting percentage. In this particular case, the Mack model yields an EC of 70,341 (see exhibit 8 bottom right corner). Assuming independence between accident years for the Mack model, this would be equal to square root of the sum of the squares of column 14, or 67,793. The percentage adjustment would therefore be 70,341 / 67,793 = 1.038.

Panning Method

The Panning methodology for the calculation of a risk margin is based mostly on incremental paid claims. Using this data eliminates correlation between development years, which allows the assumption of independence in aggregating standard deviations. The incremental paid claims are displayed in exhibit 4.

Exhibit 4

Incremental Paid Claim Amounts ($000)

Acc. Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192 (...)	228	240
1988	6 386	22 202	32 259	30 133	32 914	19 197	14 896	9 429	6 573	1 483	1 152	453	366	81	6	6 (...)	0	0
1989	8 428	27 889	39 529	34 643	29 317	22 245	14 566	8 451	3 183	5 186	766	1 321	1 049	285	94	424 (...)	0	0
1990	9 391	33 417	50 199	42 974	33 813	29 284	15 518	8 583	3 671	5 180	2 169	267	805	43	197	187 (...)	0	0
1991	9 928	45 697	54 417	41 472	34 603	19 989	13 550	9 869	4 208	2 234	253	508	18	557	41	284 (...)	0	0
1992	16 301	52 390	63 458	46 634	30 171	25 531	17 389	11 131	4 856	2 370	605	795	6	18	32	39	0	0
1993	18 874	60 451	71 409	52 027	38 679	32 473	18 999	11 704	7 178	3 119	340	223	183	545	92	0	0	0
1994	21 627	68 744	76 507	55 014	44 423	28 466	20 465	12 730	4 991	2 026	2 235	648	1 296	244	0	0	0	0
1995	24 964	74 221	82 052	68 919	51 549	42 201	19 990	9 094	7 111	3 811	1 788	796	1 137	0	0	0	0	0
1996	26 066	81 771	94 164	66 961	56 364	34 909	23 948	15 209	8 956	6 198	2 714	377	0	0	0	0	0	0
1997	26 679	82 212	96 276	78 303	60 124	41 511	29 424	20 681	9 912	7 470	1 740	0	0	0	0	0	0	0
1998	27 090	84 827	114 304	91 500	67 521	47 496	32 129	17 641	12 345	5 214	0	0	0	0	0	0	0	0
1999	28 934	93 814	133 936	96 765	66 817	45 990	36 353	14 977	7 080	0	0	0	0	0	0	0	0	0
2000	29 491	100 115	129 871	99 670	72 995	50 773	33 249	18 979	0	0	0	0	0	0	0	0	0	0
2001	28 291	103 640	138 320	101 136	75 644	54 147	33 617	0	0	0	0	0	0	0	0	0	0	0
2002	30 548	107 370	135 416	106 749	84 398	62 996	0	0	0	0	0	0	0	0	0	0	0	0
2003	30 249	89 705	111 458	92 696	71 481	0	0	0	0	0	0	0	0	0	0	0	0	0
2004	25 932	69 482	85 276	69 005	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2005	23 834	47 507	47 754	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2006	22 253	44 654	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2007	20 456	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b_1	1,00	2,97	3,69	2,80	1,95	1,38	0,85	0,39	0,24	0,11	0,06	-0,01	0,03	0,01	0,00	-0,01 (...)	0,00	0,00
$se(b_1)$	0,00	0,33	0,52	0,29	0,23	0,20	0,13	0,09	0,07	0,07	0,03	0,02	0,03	0,02	0,01	0,03 (...)	0,00	0,00
b_0	0,00	2 897,2	5 987,0	8 292,5	11 267,5	8 353,3	5 942,6	5 358,9	2 199,3	1999,6	439,6	687,1	238,4	147,6	76,0	307,2 (...)	0	0
$se(b_0)$	0,00	7 749,1	12 098,8	6 727,6	5 407,3	4 391,1	2 733,7	1 886,2	1 434,5	1372,8	622,3	294,5	465,7	237,5	86,2	275,7 (...)	0	0

Linear regressions are then performed for each development period separately according to the model:

$$Y = b_0 + b_1 * X + e;$$

where *Y* is the dependent variable vector (development period 24 to 216), *X* is the independent variable vector (development period 12), b_0 is the intercept term, b_1 is the regression parameter and *e* is the error term. Also shown in the lower part of exhibit 4 are estimated parameters b_0 and b_1 with their respective standard errors. Through the same regression process (Excel with "Dummy" variables), the projected incremental unpaid claims by development period can be obtained, that is for each accident year (i.e., lower right part of the triangle) that is presented in exhibit 5.

For example, the linear regression using Y at 36 months and X at 12 months would use 18 points and gives Y = 5,987.0 + 3.69X. Thus the 24-36 payment of AY2006 would be 5,987.0 + 3.69 * 22,253 = 88,024.

Exhibit 5

Forecast Future Paid Losses ($000)

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192 (...)	228	240
1988																		
1989																		0
1990																	0	0
1991																(...)	0	0
1992																(...)	0	0
1993																84 (...)	0	0
1994															78	52 (...)	0	0
1995														350	78	12 (...)	0	0
1996													902	359	78	-1 (...)	0	0
1997												538	918	364	79	-8 (...)	0	0
1998											1 944	535	928	368	79	-13 (...)	0	0
1999										5 295	2 047	525	975	383	79	-35 (...)	0	0
2000									9 245	5 359	2 078	522	990	387	79	-41 (...)	0	0
2001								16 358	8 958	5 222	2 011	529	959	378	79	-27 (...)	0	0
2002							31 997	17 235	9 497	5 479	2 136	516	1 017	396	79	-54 (...)	0	0
2003						50 094	31 741	17 119	9 426	5 445	2 120	518	1 009	393	79	-50 (...)	0	0
2004					61 925	44 137	28 059	15 441	8 394	4 953	1 880	542	899	358	78	1 (...)	0	0
2005				75 024	57 828	41 242	26 270	14 625	7 893	4 714	1 763	554	846	341	78	26 (...)	0	0
2006			88 024	70 598	54 740	39 061	24 922	14 011	7 516	4 534	1 676	562	805	328	78	44 (...)	0	0
2007		63 737	81 399	65 566	51 229	36 581	23 389	13 312	7 086	4 330	1 576	573	759	314	78	66 (...)	0	0
Unpaid Losses		63 737	169 423	211 187	225 721	211 115	166 379	108 101	68 015	45 331	19 229	5 913	11 008	4 721	1 098	57 (...)	0	0
Standard Error		11 638	26 324	18 442	17 699	16 705	11 998	9 506	8 333	9 249	4 873	2 719	5 194	3 314	1 517	6 256 (...)		

Total Unpaid Losses	1 311 035
Total Stand. Deviation	47 546
Coefficient of Variation	3,63%
VaR @ 90%	1 371 968
EC @ 90%	60 933

Total unpaid claims and standard deviation of the sum of projections of unpaid claims, both by development period, are shown below the forecasted claims. Standard deviations are obtained by taking the square root of the variance covariance matrix defined by the formula:

Var-Cov Matrix $= s^2 * [I + X_0 (X'X)^{-1} X_0']$;

where s^2 is the standard deviation of error terms (i.e., the difference between actual and fitted values), X_0 is the bottom part of X corresponding to the values to be projected and X_0' and X' are the transpose of X_0 and X. Exhibit 5 gives the mean, which is equal to the total unpaid claims, and the standard deviation, which corresponds to the square root of the sum of individual standard errors (both for all development periods) of the aggregate unpaid claims distribution. The EC values are then determined using that distribution's parameters, still with the Normal approximation and a confidence level of 90%:

EC $= z_\alpha *$ (Total Std Dev.);

where z_α has the same meaning as for the Hertig method.

Mack Method (Incurred)

As for the Hertig method above, the Mack model is essentially based on cumulative incurred claims, presented in exhibit 1. A distribution-free model is assumed, for which the total standard deviation of the unpaid claims is estimated. The methodology makes the distinction between process variance and parameter variance. The process variance is based on the standard errors between expected and actual claims development, i.e., between age-to-age development factors and individual LDFs, the parameter variance is calculated as:

$$ATA = (\Sigma C_j)/(\Sigma C_{j-1}) \; ;$$

Here *j* represents the "true" development period and *C* the total claims up to the number of observations in the latest development period. These factors are displayed in exhibit 6.

Exhibit 6

Loss Development Factors

Accident Year	Development Periods (months)
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (...)	216-228	228-240
1988	1,4062	1,2965	1,1499	1,0559	0,9926	1,0110	1,0111	0,9966	0,9994	0,9955	1,0022	0,9974	0,9999	1,0000 (...)	1,0000	1,0000
1989	1,4698	1,3141	1,1087	1,0212	1,0027	1,0037	1,0014	0,9978	1,0033	0,9974	1,0011	0,9985	1,0019	0,9998 (...)	1,0000	
1990	1,5021	1,2448	1,0462	1,0181	1,0019	1,0037	0,9971	1,0014	1,0035	0,9988	0,9997	1,0025	0,9991	1,0005 (...)		
1991	1,5132	1,1907	1,0496	1,0067	0,9999	1,0023	0,9853	0,9984	0,9975	0,9995	1,0051	0,9998	0,9971	1,0000 (...)		
1992	1,4011	1,1226	1,0687	0,9963	1,0060	1,0038	0,9944	1,0018	0,9996	0,9984	0,9988	1,0000	0,9991	1,0015 (...)		
1993	1,2987	1,1370	1,0198	1,0063	0,9941	0,9971	0,9998	1,0027	1,0003	0,9999	0,9999	0,9997	1,0000	0,9998		
1994	1,2988	1,1027	1,0358	0,9865	1,0038	0,9942	0,9973	1,0033	1,0004	1,0046	0,9987	1,0008	0,9999			
1995	1,2444	1,1085	1,0306	1,0018	1,0092	1,0117	1,0050	1,0022	0,9994	0,9988	1,0002	1,0011				
1996	1,2395	1,0825	1,0295	1,0313	1,0052	0,9990	1,0054	1,0041	0,9998	1,0020	0,9999					
1997	1,2116	1,0810	1,0780	1,0342	1,0090	1,0073	1,0017	1,0057	1,0029	0,9998						
1998	1,1852	1,1300	1,0654	1,0279	1,0156	1,0029	0,9984	1,0068	1,0031							
1999	1,2021	1,1144	1,0542	1,0166	1,0130	1,0057	1,0045	1,0035								
2000	1,2292	1,0978	1,0527	1,0215	1,0117	1,0109	0,9995									
2001	1,2534	1,0911	1,0406	1,0128	1,0155	1,0068										
2002	1,2535	1,0874	1,0372	1,0211	1,0105											
2003	1,1796	1,0723	1,0457	1,0166												
2004	1,0917	1,0542	1,0436													
2005	1,0820	1,0342														
2006	1,0310															
2007																
Age-to-age LDF	1,2263	1,1087	1,0505	1,0171	1,0081	1,0046	1,0004	1,0029	1,0010	0,9999	1,0004	1,0002	0,9995	1,0003 (...)	1,0000	1,0000
Cumul. Age-to-age	1,4768	1,2043	1,0862	1,0340	1,0166	1,0085	1,0039	1,0035	1,0006	0,9996	0,9997	0,9993	0,9991	0,9996 (...)	1,0000	1,0000

The ATA LDF of the development period 156-168 period is (177,536 + … + 340,223) / (177,555 + … + 340,258) = 0.9995. The cumulative ATA of the same period would be 0.9995 * 0.9996 = 0.9991. The calculation of the error terms between the actual LDF and the development period ATA are shown in Exhibit 7. For example, the 12-24 period error term for AY2006 is $(1.0310 – 1.2263)^2 * 306,406 = 11,679$.

Exhibit 7

Error Terms

Acc. Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	(...)	216-228	228-240
													Development Periods (months)				
1988	2582	3955	1435	252	42	7	20	7	0	3	1	1	0	0	(...)	0	0
1989	5319	5560	586	3	6	0	0	5	1	1	0	1	1	0	(...)	0	
1990	8948	3270	4	0	9	0	2	1	1	0	0	1	0	0	(...)		
1991	10435	1290	0	26	16	1	55	5	3	0	5	0	1	0	(...)		
1992	4953	44	84	118	1	0	10	0	1	1	1	0	0	0	(...)		
1993	1108	220	293	37	63	18	0	0	0	0	0	0	0	0			
1994	1221	11	72	321	6	37	3	0	0	8	1	0	0				
1995	87	0	145	88	0	19	8	0	1	0	0	0					
1996	51	246	173	82	3	13	11	1	1	2	0						
1997	65	285	302	127	0	3	1	4	2	0							
1998	573	182	101	57	28	2	2	8	2								
1999	214	15	7	0	13	1	9	0									
2000	3	55	2	11	7	22	0										
2001	295	155	54	10	32	3											
2002	319	244	105	10	4												
2003	902	647	12	0													
2004	6730	1206	21														
2005	5961	1719															
2006	11679																
2007																	
S-square	3414	1124	212	76	16	10	10	3	1	2	1	1	0	0	(...)	0	0

Process Variance Multiplier

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	(...)	216-228	228-240
Incremental	3353	1101	209	76	16	10	10	3	1	2	1	1	0	0	(...)	0	0
Cumulative	4783	1430	329	120	44	28	18	8	5	4	2	1	1	0	(...)	0	0

Parameter Variance Multiplier

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	(...)	216-228	228-240
Incremental	0,000439	0,000152	0,000030	0,000012	0,000003	0,000002	0,000002	0,000001	0,000000	0,000001	0,000000	0,000000	0,000000	0,000000	(...)	0,000000	0,000000
Cumulative	0,000642	0,000203	0,000052	0,000022	0,000010	0,000007	0,000005	0,000003	0,000002	0,000002	0,000001	0,000001	0,000001	0,000000	(...)	0,000000	0,000000

The S-square is equal to the sum of the error term divided by j-1, while the incremental process variance multiplier is equal to the S-square * Cumulative ATA / ATA^2. For the period 12-24, this would yield $3{,}414 * 1.4768/1.2263^2 = 3{,}353$.

The parameter variance multiplier is equal to the $(S\text{-square}/ATA^2)$ / (the sum of the cumulative incurred claims for the development period, up to j-1). For the period 12-24, this would give $(3{,}414/1.2263^2)$ / $(79{,}766 + 89{,}661 + \ldots + 306{,}406) = 0.000439$.

Exhibit 8 summarizes the steps and calculations to get to EC values (in this particular case, still at a confidence level of 90% and still using a normal approximation for the value at risk measures) so that:

$EC = z_\alpha *$(Total Std Dev.).

Exhibit 8

Acc. Year (1)	Diagonal (2)	LDF (3)	Reserve (4)=(5)-(2)	Ultimate (5) = (3)*(2)	Proc Mult (6)	Proc SD (7)	CV (8)=(7)/(4)	Param SD (9)	CV (10)=(9)/(4)	Total SD (11)	CV (12)=(11)/(4)	Var @ 90% (13)	EC @ 90% (14)=(13)-(4)
1988	177 538	1.0000	0	177 538									
1989	197 375	1.0000	0	197 375	0	0	0%	0	0%	0	0%	0	0
1990	235 697	1.0000	0	235 697	0	0	0%	0	0%	0	0%	0	0
1991	237 754	0.9993	-164	237 590	0	229	-140%	143	-87%	270	-165%	183	347
1992	272 282	0.9993	-181	272 101	0	246	-136%	164	-91%	296	-163%	198	380
1993	317 444	0.9993	-213	317 231	0	285	-134%	199	-93%	348	-163%	232	445
1994	340 223	0.9996	-134	340 088	0	359	-267%	235	-175%	429	-319%	415	550
1995	388 036	0.9991	-335	387 702	1	569	-170%	333	-99%	659	-197%	510	845
1996	420 492	0.9993	-288	420 205	1	761	-265%	418	-145%	868	-302%	825	1 113
1997	458 419	0.9997	-139	458 280	2	1 044	-751%	537	-386%	1 174	-845%	1 366	1 505
1998	511 894	0.9996	-211	511 683	4	1 447	-686%	712	-338%	1 613	-765%	1 856	2 067
1999	540 779	1.0006	310	541 089	5	1 696	548%	817	264%	1 882	608%	2 722	2 412
2000	563 478	1.0035	1 961	565 440	8	2 137	109%	972	50%	2 347	120%	4 969	3 008
2001	589 923	1.0039	2 275	592 197	18	3 289	145%	1 343	59%	3 552	156%	6 827	4 552
2002	628 131	1.0085	5 335	633 466	28	4 208	79%	1 674	31%	4 529	85%	11 139	5 804
2003	556 491	1.0166	9 263	565 754	44	5 014	54%	1 770	19%	5 317	57%	16 077	6 814
2004	446 242	1.0340	15 177	461 419	120	7 456	49%	2 143	14%	7 758	51%	25 119	9 942
2005	320 453	1.0862	27 634	348 088	329	10 708	39%	2 505	9%	10 997	40%	41 728	14 093
2006	315 912	1.2043	64 543	380 456	1 430	23 328	36%	5 426	8%	23 951	37%	95 238	30 694
2007	264 657	1.4768	126 187	390 844	4 783	43 237	34%	9 903	8%	44 356	35%	183 032	56 845
	7 783 221		251 019			51 472	21%	19 059	8%	54 887	22%	321 360	70 341

(2) & (3) from Exhibit 1 and 6

(6) from Exhibit 7

(7) = Square root [(5)*(6)], with the total being the square root of the sum of the squares

(9) = [Ult. Losses]*Square root[Cumul. Param. Var. Mult.], while the total is the square root of the sum { [Ult. Losses]$_i$ * [Ult.Losses]$_j$ *

OR ([Cumul.Param.Var. Mult]$_i$ for i>j , [Cumul.Param.Var. Mult]$_j$ for j>i) }

e.g. [Ult. Losses]$_{36}$ * [Ult. Losses]$_{60}$ * [Cumul.Param.Var.Mult]$_{60-72}$

(11) = Square root of the sum of Variances [(7) and (9) squared]

(13) = $\mu + z_\alpha * \sigma$ [Normal approximation]

Mack Method (Paid)

A second version of the Mack method works with cumulative paid claims. These are displayed in Exhibit 9.

Cumulative Paid Claim Amounts ($000)

Acc. Year	Development Level (months)																
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180 (…)	228	240
1988	6 386	28 588	60 847	90 980	123 894	143 092	157 988	167 417	173 991	175 474	176 626	177 079	177 445	177 526	177 532 (…)	177 538	177 538
1989	8 428	36 317	75 846	110 490	139 806	162 052	176 618	185 069	188 251	193 437	194 202	195 523	196 572	196 857	196 951 (…)	197 375	
1990	9 391	42 808	93 007	135 982	169 794	199 078	214 596	223 179	226 850	232 030	234 199	234 466	235 271	235 314	235 510 (…)		
1991	9 928	55 625	110 043	151 515	186 118	206 107	219 657	229 526	233 735	235 968	236 221	236 729	236 747	237 304	237 345 (…)		
1992	16 301	68 691	132 149	178 783	208 954	234 485	251 875	263 006	267 862	270 232	270 837	271 632	271 638	271 656	271 688 (…)		
1993	18 874	79 325	150 734	202 761	241 440	273 914	292 913	304 616	311 794	314 913	315 253	315 476	315 659	316 204	316 296		
1994	21 627	90 372	166 879	221 894	266 317	294 783	315 248	327 979	332 970	334 996	337 231	337 880	339 175	339 419			
1995	24 964	99 184	181 237	250 156	301 705	343 906	363 897	372 991	380 102	383 912	385 700	386 496	387 633				
1996	26 066	107 838	202 002	268 963	325 327	360 236	384 184	399 393	408 349	414 548	417 262	417 638					
1997	26 679	108 891	205 167	283 469	343 594	385 104	414 528	435 209	445 121	452 591	454 331						
1998	27 090	111 917	226 221	317 721	385 243	432 739	464 868	482 509	494 854	500 068							
1999	28 934	122 749	256 685	353 449	420 266	466 257	502 610	517 586	524 666								
2000	29 491	129 606	259 477	359 147	432 142	482 915	516 164	535 143									
2001	28 291	131 931	270 250	371 387	447 031	501 178	534 794										
2002	30 548	137 918	273 334	380 082	464 481	527 476											
2003	30 249	119 954	231 412	324 107	395 588												
2004	25 932	95 413	180 689	249 694													
2005	23 834	71 341	119 095														
2006	22 253	66 908															
2007	20 456																

The methodology with paid claims is essentially the same as for that with incurred claims. Exhibits 6, 7, and 8 are replicated with paid data under exhibits 10 to 12 and formulas shown above are unchanged. Implications of using incurred versus paid claims as well as the different results from both methods are discussed below in the next section.

Exhibit 10

Loss Development Factors

Acc. Year	Development Periods (months)																
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (…)	216-228	228-240
1988	4,4766	2,1284	1,4952	1,3618	1,1550	1,1041	1,0597	1,0393	1,0085	1,0066	1,0026	1,0021	1,0005	1,0000 (…)	1,0000	1,0000
1989	4,3092	2,0884	1,4568	1,2653	1,1591	1,0899	1,0478	1,0172	1,0275	1,0040	1,0068	1,0054	1,0014	1,0005 (…)	1,0000	
1990	4,5583	2,1726	1,4621	1,2487	1,1725	1,0779	1,0400	1,0164	1,0228	1,0093	1,0011	1,0034	1,0002	1,0008 (…)		
1991	5,6027	1,9783	1,3769	1,2284	1,1074	1,0657	1,0449	1,0183	1,0096	1,0011	1,0021	1,0001	1,0024	1,0002 (…)		
1992	4,2140	1,9238	1,3529	1,1688	1,1222	1,0742	1,0442	1,0185	1,0088	1,0022	1,0029	1,0000	1,0001	1,0001 (…)		
1993	4,2029	1,9002	1,3452	1,1908	1,1345	1,0694	1,0400	1,0236	1,0100	1,0011	1,0007	1,0006	1,0017	1,0003		
1994	4,1786	1,8466	1,3297	1,2002	1,1069	1,0694	1,0404	1,0152	1,0061	1,0067	1,0019	1,0038	1,0007			
1995	3,9731	1,8273	1,3803	1,2061	1,1399	1,0581	1,0250	1,0191	1,0100	1,0047	1,0021	1,0029				
1996	4,1371	1,8732	1,3315	1,2096	1,1073	1,0665	1,0396	1,0224	1,0152	1,0065	1,0009					
1997	4,0815	1,8842	1,3817	1,2121	1,1208	1,0764	1,0499	1,0228	1,0168	1,0038						
1998	4,1313	2,0213	1,4045	1,2125	1,1233	1,0742	1,0379	1,0256	1,0105							
1999	4,2423	2,0911	1,3770	1,1890	1,1094	1,0780	1,0298	1,0137								
2000	4,3947	2,0020	1,3841	1,2032	1,1175	1,0689	1,0368									
2001	4,6634	2,0484	1,3742	1,2037	1,1211	1,0671										
2002	4,5148	1,9819	1,3905	1,2221	1,1356											
2003	3,9656	1,9292	1,4006	1,2205												
2004	3,6794	1,8938	1,3819													
2005	2,9932	1,6694														
2006	3,0066															
2007																
	4,1067	1,9500	1,3819	1,2127	1,1250	1,0722	1,0394	1,0205	1,0128	1,0046	1,0021	1,0023	1,0010	1,0003 (…)	1,0000	1,0000
	17,5857	4,2822	2,1960	1,5891	1,3105	1,1648	1,0863	1,0451	1,0242	1,0112	1,0066	1,0045	1,0022	1,0012 (…)	1,0000	1,0000

Error Terms

Acc. Year	Development Periods (months)															
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (...)	216-228	228-240
1988	874	910	782	2023	111	145	65	59	3	1	0	0	0	0 (...)	0	0
1989	346	696	425	307	162	50	13	2	41	0	4	2	0	0 (...)	0	
1990	1916	2121	598	176	382	6	0	4	23	5	0	0	0	0 (...)		
1991	22219	44	3	37	58	9	7	1	2	3	0	1	0	0 (...)		
1992	188	47	111	345	2	1	6	1	4	1	0	1	0	0 (...)		
1993	175	197	203	97	22	2	0	3	2	4	1	1	0	0		
1994	112	967	455	34	88	2	0	9	15	1	0	1	0			
1995	445	1495	0	11	66	69	76	1	3	0	0	0				
1996	24	637	513	3	102	12	0	2	2	2	1					
1997	17	473	0	0	6	7	46	2	7	0						
1998	16	569	116	0	1	2	1	13	3							
1999	532	2444	6	197	102	15	46	24								
2000	2447	350	1	32	25	6	4									
2001	8768	1277	16	30	7	13										
2002	5087	140	21	34	52											
2003	603	52	81	20												
2004	4735	302	0													
2005	29550	5620														
2006	26930															
2007																
S-square	5832	1079	208	223	85	26	22	11	11	2	1	1	0	0 (...)	0	0

Process Variance Multiplier

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (...)	216-228	228-240
Incremental	6082	1215	239	241	88	26	22	11	11	2	1	1	0	0 (...)	0	0
Cumulative	7939	1857	642	403	162	74	48	26	15	4	2	1	0	0 (...)	0	0

Parameter Variance Multiplier

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180 (...)	216-228	228-240
Incremental	0,000833	0,000173	0,000035	0,000038	0,000015	0,000005	0,000005	0,000003	0,000003	0,000001	0,000000	0,000000	0,000000	0,000000 (...)	0,000000	0,000000
Cumulative	0,001111	0,000279	0,000106	0,000070	0,000032	0,000017	0,000012	0,000007	0,000005	0,000002	0,000001	0,000001	0,000000	0,000000 (...)	0,000000	0,000000

Exhibit 12

Acc. Year	Diagonal	LDF	Reserve	Ultimate	Proc Mult	Proc SD	CV	Param SD	CV	Total SD	CV	VaR @ 90%	EC @ 90%
(1)	(2)	(3)	(4)=(5)-(2)	(5) = (3)*(2)	(6)	(7)	(8)=(7)/(4)	(9)	10)=(9)/(4)	(11)	12)=(11)/(4)	(13)	(14)=(13)-(4)
1988	177 538	1,0000	0	177 538									
1989	197 375	1,0000	0	197 375	0	0	0%	0	0%	0	0%	0	0
1990	235 697	1,0000	0	235 697	0	0	0%	0	0%	0	0%	0	0
1991	237 629	1,0000	0	237 629	0	0	-164%	0	-102%	0	-193%	0	0
1992	271 727	1,0003	95	271 823	0	133	140%	75	79%	152	160%	290	195
1993	316 296	1,0009	293	316 589	0	261	89%	145	50%	299	102%	676	383
1994	339 419	1,0012	423	339 843	0	283	67%	161	38%	326	77%	841	417
1995	387 633	1,0022	872	388 505	0	404	46%	223	26%	462	53%	1 464	592
1996	417 638	1,0045	1 883	419 521	1	757	40%	367	19%	841	45%	2 961	1 078
1997	454 331	1,0066	3 006	457 337	2	983	33%	470	16%	1 089	36%	4 402	1 396
1998	500 068	1,0112	5 612	505 680	4	1 435	26%	659	12%	1 579	28%	7 635	2 023
1999	524 666	1,0242	12 672	537 337	15	2 805	22%	1 162	9%	3 036	24%	16 563	3 891
2000	535 143	1,0451	24 152	559 295	26	3 782	16%	1 517	6%	4 075	17%	29 374	5 223
2001	534 794	1,0863	46 163	580 957	48	5 258	11%	2 020	4%	5 633	12%	53 381	7 219
2002	527 476	1,1648	86 930	614 406	74	6 745	8%	2 545	3%	7 209	8%	96 168	9 239
2003	395 588	1,3105	122 812	518 400	162	9 157	7%	2 941	2%	9 618	8%	135 137	12 326
2004	249 694	1,5891	147 102	396 796	403	12 642	9%	3 322	2%	13 071	9%	163 854	16 751
2005	119 095	2,1960	142 433	261 527	642	12 959	9%	2 687	2%	13 235	9%	159 394	16 961
2006	66 908	4,2822	219 606	286 514	1 857	23 067	11%	4 783	2%	23 557	11%	249 796	30 190
2007	20 456	17,5857	339 281	359 737	7 939	53 440	16%	11 993	4%	54 770	16%	409 471	70 190
	6 509 174		1 153 333			62 440	5%	23 012	2%	66 546	6%	1 238 615	85 282

(2) & (3) from Exhibit 1 and 6

(6) from Exhibit 7

(7) = Square root [(5)*(6)]

(9) = [Ult. Losses]*Square root[Cumul. Param. Var. Mult.]

(11) = Square root of the sum of Variances [(7) and (9) squared]

(13) = μ + z_α*σ [Normal approximation]

Results

Finally, exhibit 13 shows a summary of results provided by the three methods. Note that these results exclude diversification benefits since there was only one line of business analyzed.

Exhibit 13

Summary

Methods	Data Used	Mean Reserves	Standard Deviation	Coefficient of Variation	VaR @ 90%	EC @ 90%
Hertig	Cumulative Incurred	1 543 757	n/a	n/a	1 629 968	86 211
Panning	Incremental Paid	1 311 035	47 546	3,6%	1 371 968	60 933
Mack	Cumulative Incurred	1 525 066	54 887	3,6%	1 595 407	70 341
Mack	Cumulative Paid	1 153 333	66 546	5,8%	1 238 615	85 282

Here are some comments concerning the different outputs:

The Hertig method does not use the VaR measure since it quantifies the volatility of incurred claims, not the volatility of the unpaid claims estimate as the other methods do. The number representing the VaR for Hertig method in exhibit 13 is the implied number given the EC and the mean unpaid claims.

The EC results are quite sparse, with the Hertig and the Mack paid methods providing the highest results. The Mack incurred has a relatively higher required EC than the Panning method, but they have an equal coefficient of variation, lower than the one from the Mack paid.

There is one last remark regarding the Mack method. Even though aggregate EC results using incurred claims seem normal, this methodology exhibits large negative coefficients of variation for older accident years caused by negative development. The approach based on paid claims seems to produce more stable results, but still shows quite large coefficients of variation, also in older accident years. However, if negative development occurs later in the development, there might be some inconsistencies in estimating the variation in the tail. In this case, the use of paid claims data might be a better choice.